22nd ZIMÁNYI SCHOOL WINTER WORKSHOP ON HEAVY ION PHYSICS
December 5-9, 2022, Budapest, Hungary

Production and Fragmentation of Heavy Flavor in Small Systems

(connection of the hard process to event shapes and the underlying event)

Róbert Vértesi
for the **ALICE** collaboration
vertesi.robert@wigner.hu

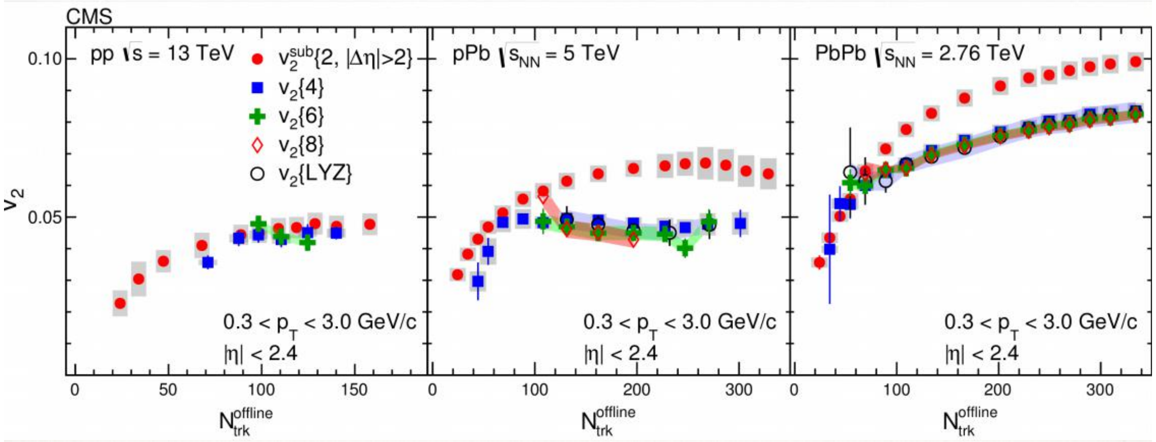
This work has been supported by the Hungarian NKFIH OTKA FK131979 and K135515
as well as the NKFIH 2019-2.1.11-TÉT-2019-00078, 2019-2.1.11-TÉT-2019-00050, 2019-2.1.6-NEMZKI-2019-00011 grants

Collectivity

- **RHIC:** Hot nuclear matter behaves as a perfect fluid (very low viscosity)
  => strongly coupled quark-gluon plasma (sQGP)

- **LHC: Collectivity in small colliding systems (pp, p-Pb)**,
  in case of high event activity (eg. final-state multiplicity)

CMS, Phys. Lett. B 765 (2017) 193

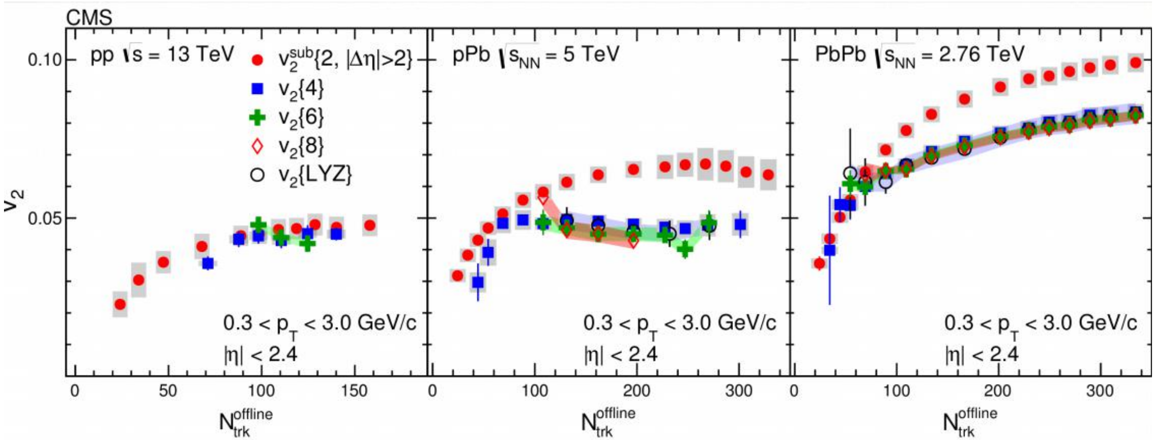


Collectivity

- **RHIC:** Hot nuclear matter behaves as a perfect fluid (very low viscosity)
  => strongly coupled quark-gluon plasma (sQGP)

- **LHC: Collectivity in small colliding systems (pp, p-Pb)**, in case of high event activity (eg. final-state multiplicity)

CMS, Phys. Lett. B 765 (2017) 193



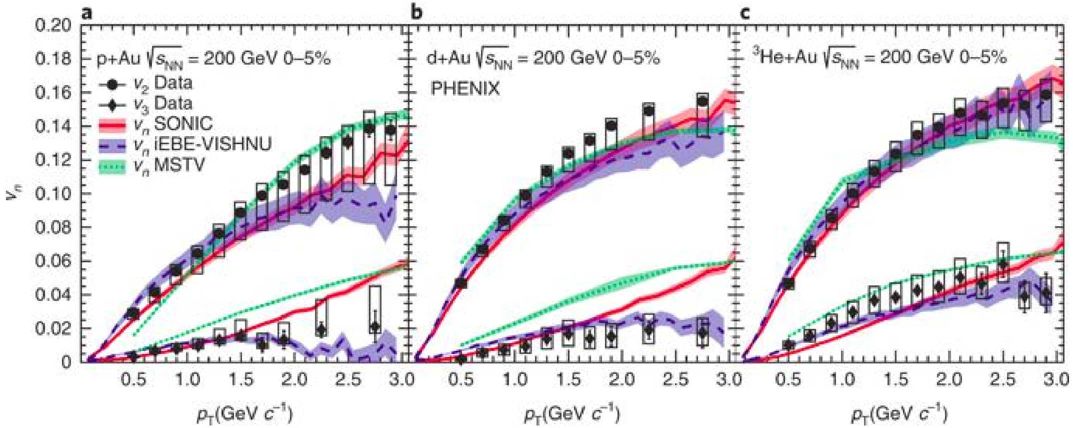
- **Collectivity** = long-range multiparticle correlations (eg. $v_n\{n\}$)
  does not necessarily require hydrodynamical evolution!

Hydrodynamical evolution

- **PHENIX 2018**: Comparison of p-Au, d-Au, He-Au collisions prefer hydrodinamics compared to models with color flux tubes

$$v_2^{\mathrm{p+Au}} < v_2^{\mathrm{d+Au}} \approx v_2^{\mathrm{^3He+Au}}$$
$$v_3^{\mathrm{p+Au}} \approx v_3^{\mathrm{d+Au}} < v_3^{\mathrm{^3He+Au}}$$

Nature Physics 15, 214 (2019)

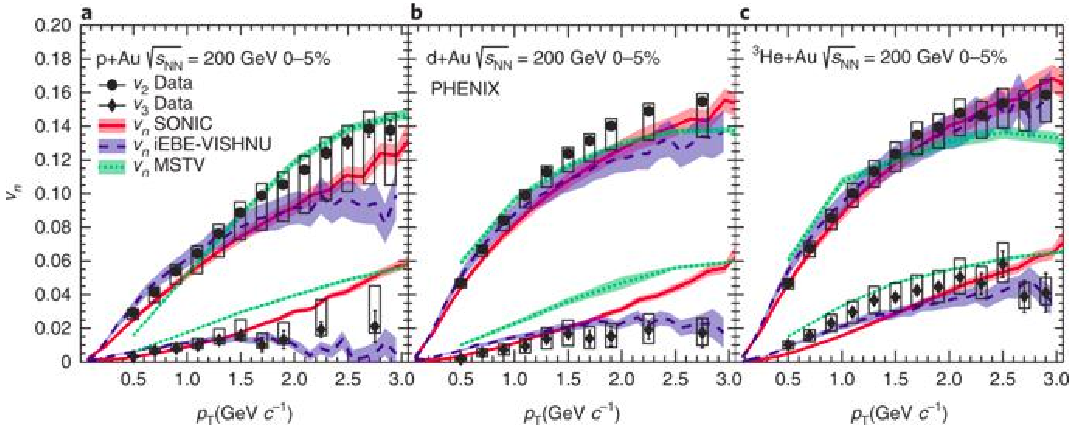


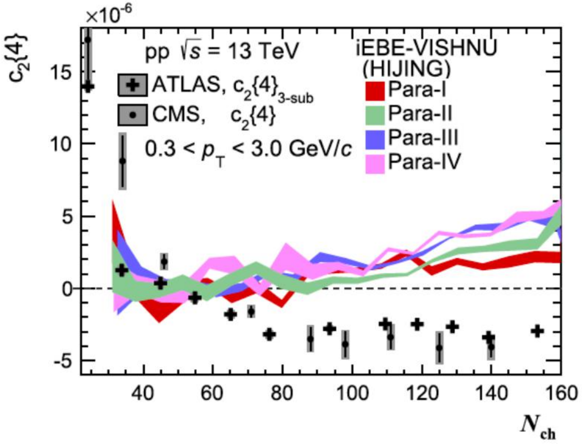
Hydrodynamical evolution

- **PHENIX 2018**: Comparison of p-Au, d-Au, He-Au collisions prefer hydrodinamics compared to models with color flux tubes

$$v_2^{p+Au} < v_2^{d+Au} \approx v_2^{^3He+Au}$$
$$v_3^{p+Au} \approx v_3^{d+Au} < v_3^{^3He+Au}$$

Nature Physics 15, 214 (2019)





- Sign of cumulant differs from that predicted by hydrodynamics => **?**

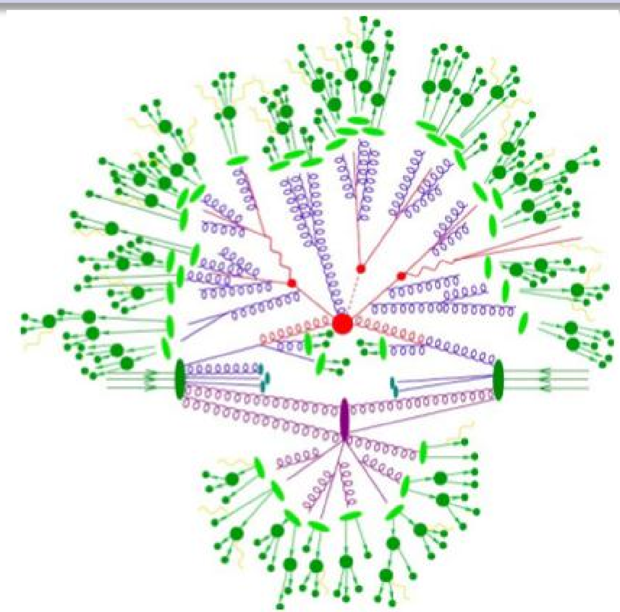
W. Zhao et al., PLB 780 (2018) 495

$$v_n\{2\} = \sqrt{c_n\{2\}}$$
$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}$$
...

Hot QGP or cold QCD?

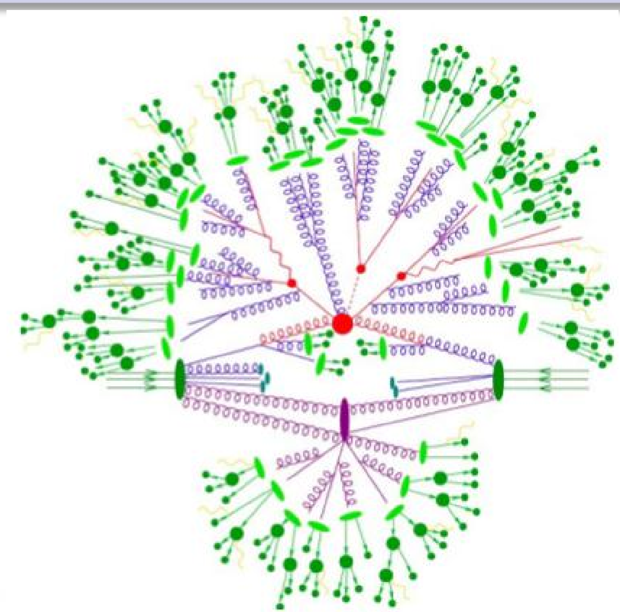
- **Does QGP come into being in small systems?**
  - Is energy density high enough?
  - Can hot matter thermalize?

- Current understanding:
  - **Collective behavior does not require QGP**
  - Can be produced by vacuum-QCD effects at the soft-hard boundary



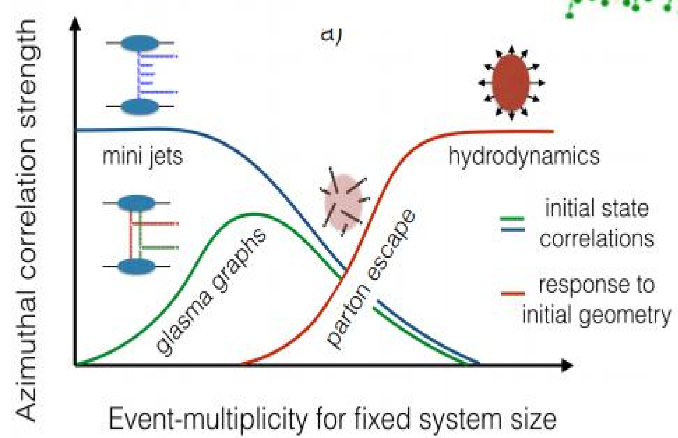
Hot QGP or cold QCD?

- **Does QGP come into being in small systems?**
  - Is energy density high enough?
  - Can hot matter thermalize?



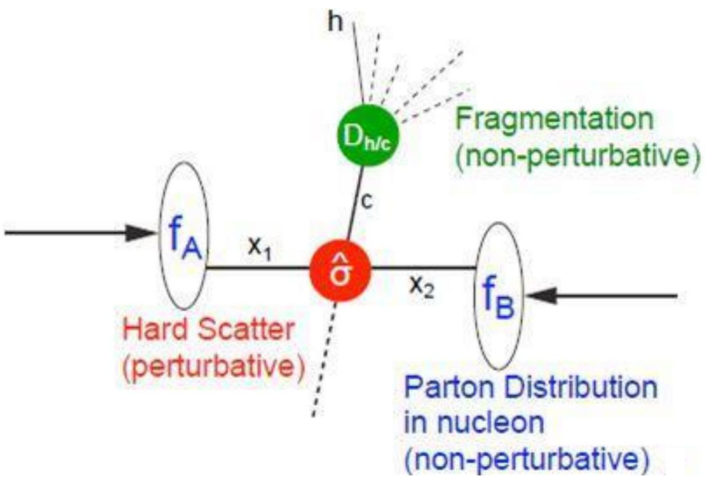
- Current understanding:
  - **Collective behavior does not require QGP**
  - Can be produced by vacuum-QCD effects at the soft-hard boundary

  - **Multi-parton interactions** (MPI)
    eg. **Schlichting, arXiv:1601.01177**
  - **Color-reconnection** (part of some MPI models)
    eg. **Ortiz-Bencédi-Bello, J.Phys.G 44 (2017)**
  - **Minijets** (semihard partons produced by incoming partons or bremsstrahlung)
    eg. **Eskola, Nucl.Part.Phys. 22, 4, 185 (1998)**



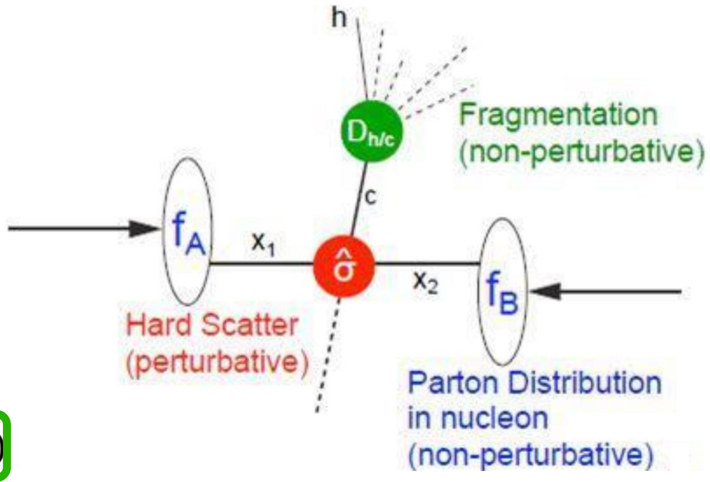
Heavy flavor: production and fragmentation

- **Production of heavy-flavor hadrons:**
  - Parton distribution functions (PDF)
  - Hard scattering process
  - Fragmentation



Heavy flavor: production and fragmentation

- **Production of heavy-flavor hadrons:**
  - Parton distribution functions (PDF)
  - Hard scattering process
  - Fragmentation
- Factorization hypothesis: these 3 are independent!

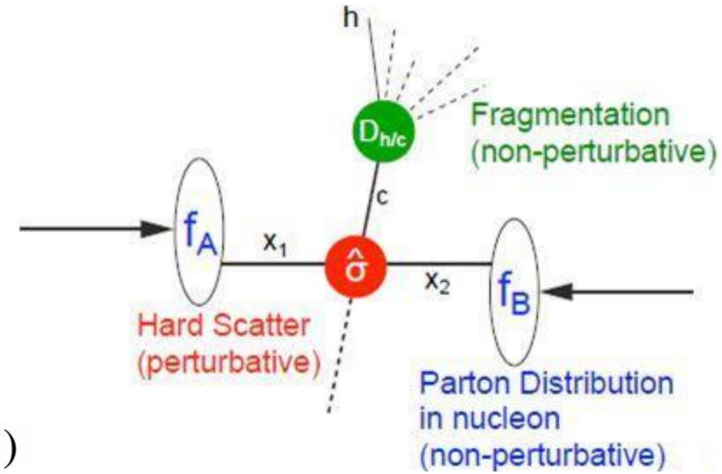


$$\sigma_{hh\to H} = f_a(x_1,Q^2)\otimes f_b(x_2,Q^2)\otimes\sigma_{ab\to q\bar{q}}\otimes D_{q\to H}(z_q,Q^2)$$

Feynman-*x*:
$x_i = p^A_{\|} / p^A_{\|,max}$
Q : impulzusátadás

Heavy flavor: production and fragmentation

- **Production of heavy-flavor hadrons:**
  - Parton distribution functions (PDF)
  - Hard scattering process
  - Fragmentation
- Factorization hypothesis: these 3 are independent!



$$\sigma_{hh\to H} = f_a(x_1,Q^2)\otimes f_b(x_2,Q^2)\otimes \sigma_{ab\to q\bar{q}} \otimes D_{q\to H}(z_q,Q^2)$$

Feynman-*x*:
$x_i = p^A_{||} / p^A_{||,max}$
Q : impulzusátadás

- Fragmentation functions are traditionally treated as universal (same across all colliding systems)

Heavy flavor is different

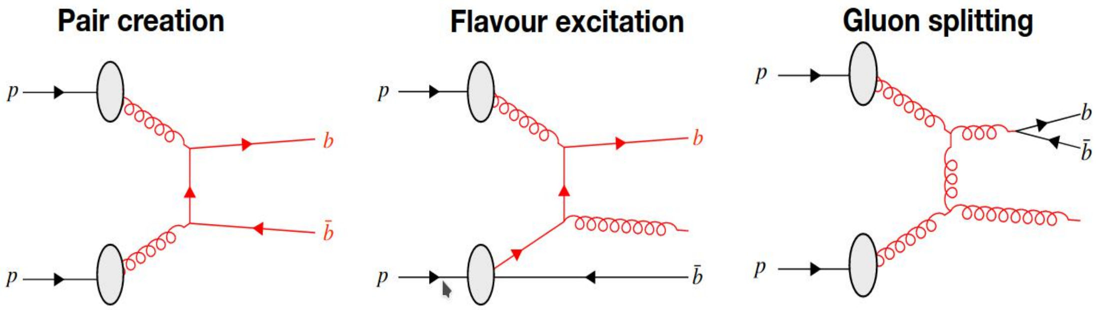
- Production mechanisms:

  $m_{c,b} >> \Lambda_{QCD}$

  → Perturbative down to low momenta

  - **LO**: Flavor creation (FLC)
  - **NLO**: Gluon splitting (GSP)

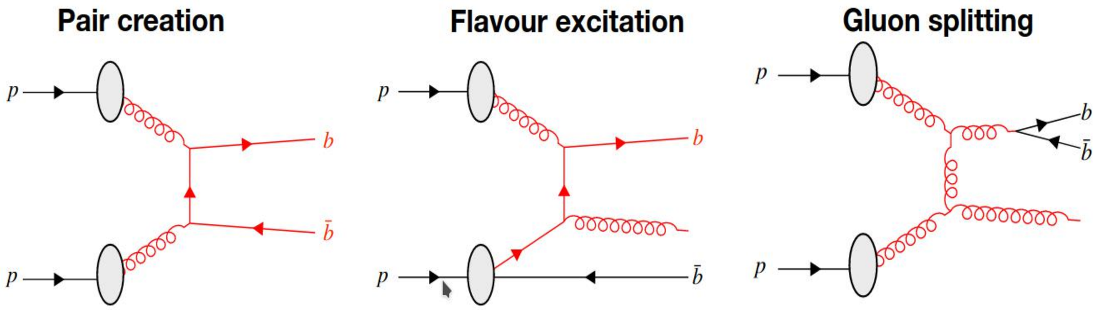
    flavor excitation (FEX)

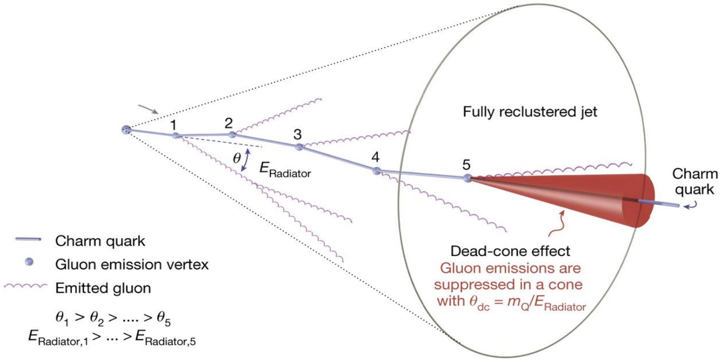


Heavy flavor is different

- Production mechanisms:
  $m_{c,b} >> \Lambda_{QCD}$
  → Perturbative down to low momenta
  - **LO**: Flavor creation (FLC)
  - **NLO**: Gluon splitting (GSP)
    flavor excitation (FEX)

- Fragmentation
  - **Color-charge effect**
    heavy flavor: quarks
    light flavor: mostly gluons
  - **Dead cone**: hard fragmentation
    No gluon radiation at small angles



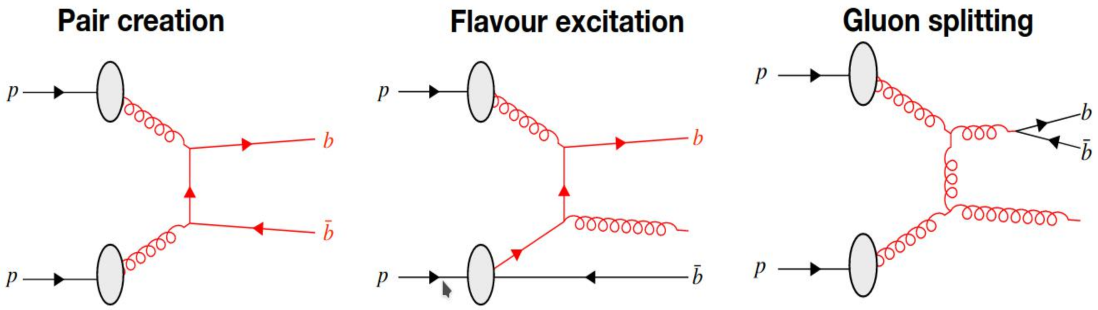


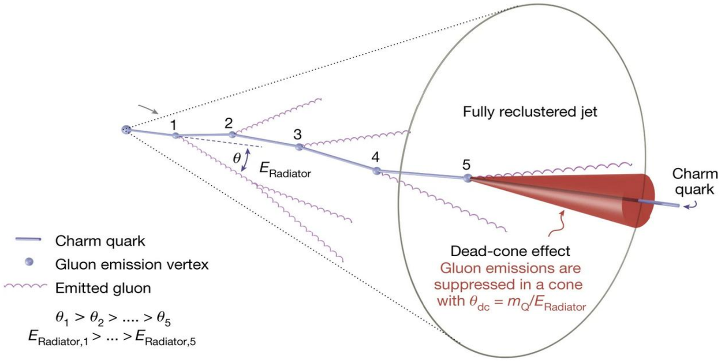
ALICE, Nature 605 (2022) 440

Heavy flavor is different

- Production mechanisms:
  $m_{c,b} >> \Lambda_{QCD}$
  → Perturbative down to low momenta
  - **LO**: Flavor creation (FLC)
  - **NLO**: Gluon splitting (GSP)
    flavor excitation (FEX)

- Fragmentation
  - **Color-charge effect**
    heavy flavor: quarks
    light flavor: mostly gluons
  - **Dead cone**: hard fragmentation
    No gluon radiation at small angles



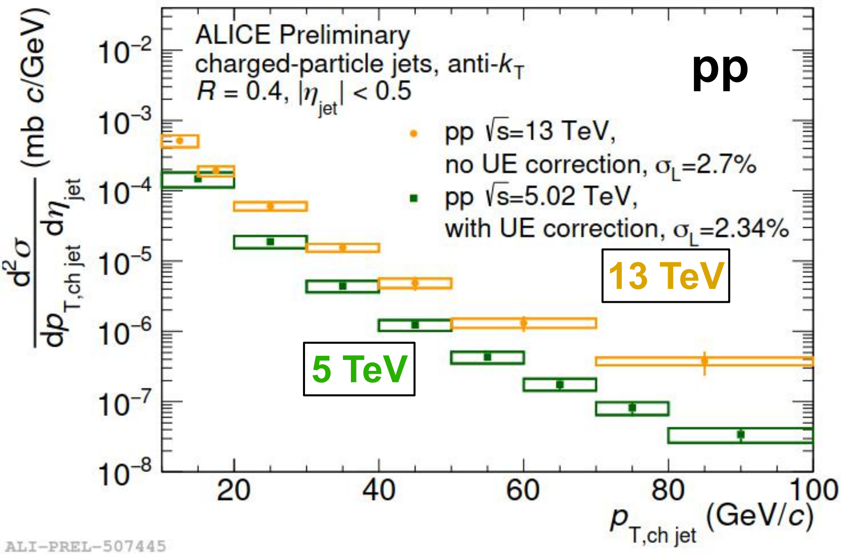


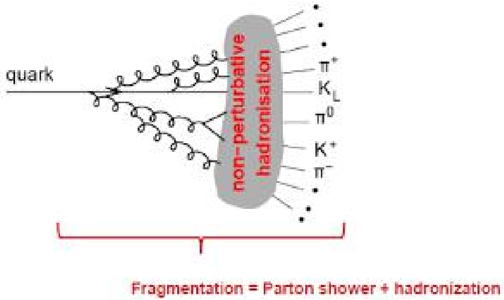
ALICE, Nature 605 (2022) 440

Systematic studies using {
event shape dependence
jet structure and correlation
connection to the underlying event

Heavy-flavor b-jet production in small systems

ALICE, JHEP 01 (2022) 178

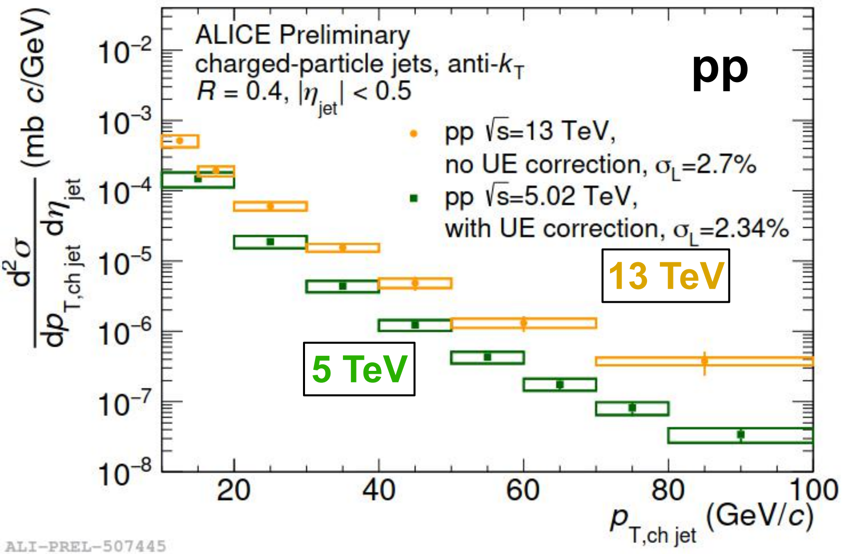


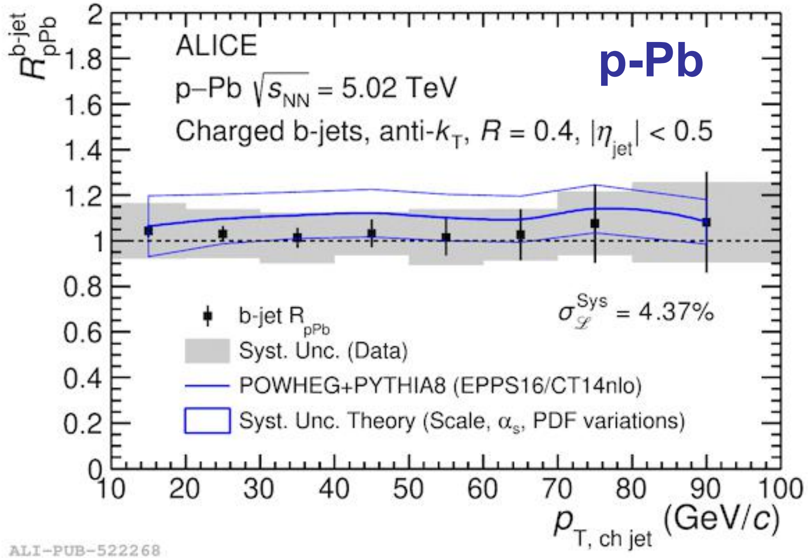


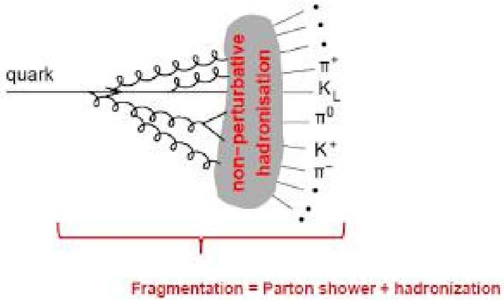
- **pp:** production of jets: input for pQCD models

Heavy-flavor b-jet production in small systems

ALICE, JHEP 01 (2022) 178







- **pp:** production of jets: input for pQCD models
- **p-Pb: Nuclear modification factor R_{pA}~1:**

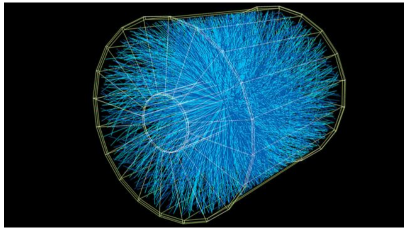
=> No strong nuclear modification present in p-A

Predicted by models assuming cold nuclear matter only

Categorizing events by activity

Charged-hadron multiplicity N_{ch} : $|\eta|<1$ N_{fw} : $2<|\eta|<5$

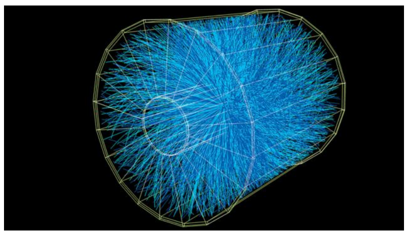
- Number of final-state charged hadrons
- Global parameter, does not take leading process into account

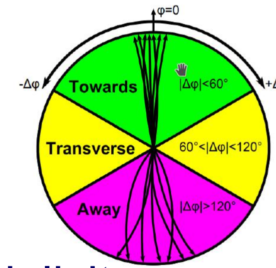


Categorizing events by activity

Charged-hadron multiplicity $N_{ch}: |\eta|<1 \quad N_{fw}: 2<|\eta|<5$

- Number of final-state charged hadrons
- Global parameter, does not take leading process into account





$$R_T = \frac{N_{trans}}{\langle N_{trans} \rangle}$$

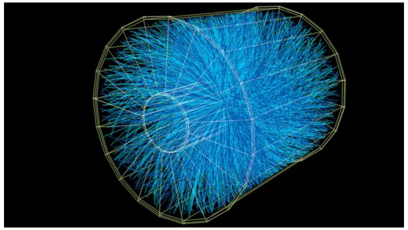
Relative transverse multiplicity

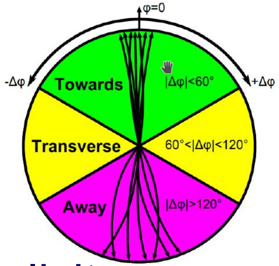
- Represents the underlying-event
- High-p_T trigger hadron => Statistics can be a problem
- Dependence on fragmentation

Categorizing events by activity

Charged-hadron multiplicity N_{ch} : $|\eta|<1$ N_{fw} : $2<|\eta|<5$

- Number of final-state charged hadrons
- Global parameter, does not take leading process into account





$$R_T = \frac{N_{trans}}{\langle N_{trans} \rangle}$$

Relative transverse multiplicity

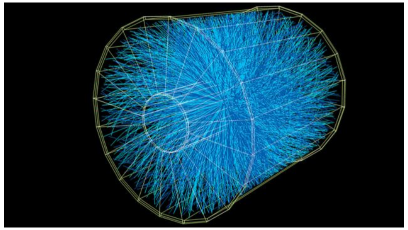
- Represents the underlying-event
- High-p_T trigger hadron => Statistics can be a problem
- Dependence on fragmentation

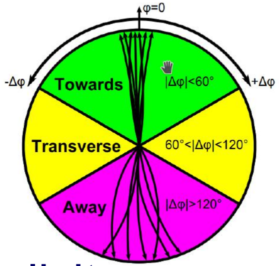
László Gyulai
Defining the UE w.r.t. HF processes
Monday 15:45 (this section)

Categorizing events by activity

Charged-hadron multiplicity N_{ch} : $|\eta|<1$ N_{fw} : $2<|\eta|<5$

- Number of final-state charged hadrons
- Global parameter, does not take leading process into account





$$R_T = \frac{N_{trans}}{\langle N_{trans} \rangle}$$

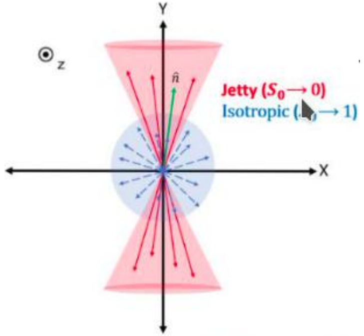
Relative transverse multiplicity

- Represents the underlying-event
- High-p_T trigger hadron => Statistics can be a problem
- Dependence on fragmentation

Transverse spherocity

- Tells apart isotropic and jetty events
- No need for trigger
- Only mid-rapidity

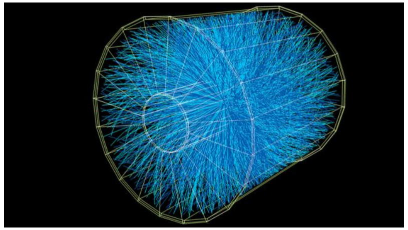
$$S_0 = \frac{\pi^2}{4}\left(\frac{\Sigma_i |\vec{p_{T_i}} \times \vec{n}|}{\Sigma_i \vec{p_{T_i}}}\right)^2$$

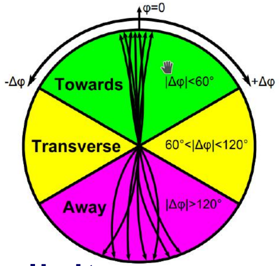


Categorizing events by activity

Charged-hadron multiplicity N_{ch} : $|\eta|<1$ N_{fw} : $2<|\eta|<5$

- Number of final-state charged hadrons
- Global parameter, does not take leading process into account





$$R_T = \frac{N_{trans}}{\langle N_{trans} \rangle}$$

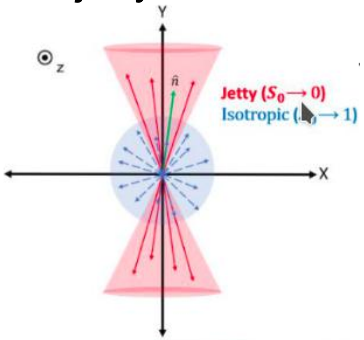
Relative transverse multiplicity

- Represents the underlying-event
- High-p_T trigger hadron => Statistics can be a problem
- Dependence on fragmentation

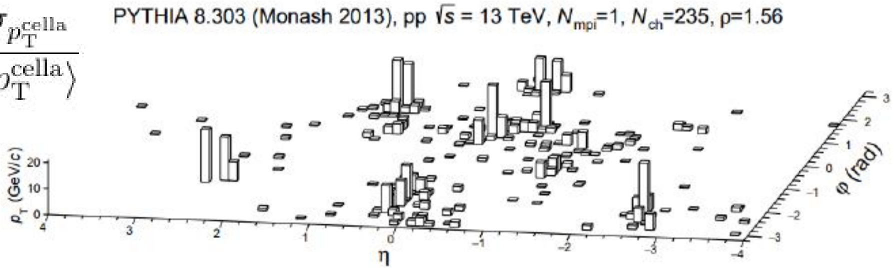
Transverse spherocity

- Tells apart isotropic and jetty events
- No need for trigger
- Only mid-rapidity

$$S_0 = \frac{\pi^2}{4} \left(\frac{\Sigma_i |\vec{p_{T_i}} \times \vec{n}|}{\Sigma_i \vec{p_{T_i}}} \right)^2$$



$$\rho = \frac{\sigma_{p_T^{cella}}}{\langle p_T^{cella} \rangle}$$



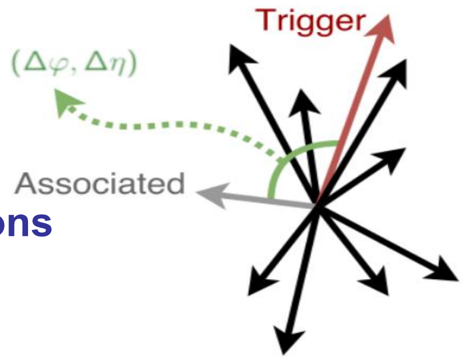
Flattenicity

A. Ortiz, G. Paic, arXiv:2204.13733

- Isotropic/hedgehog-like vs. jetty
- No need for trigger
- Full rapidity window (eg. $|\eta|<4$)

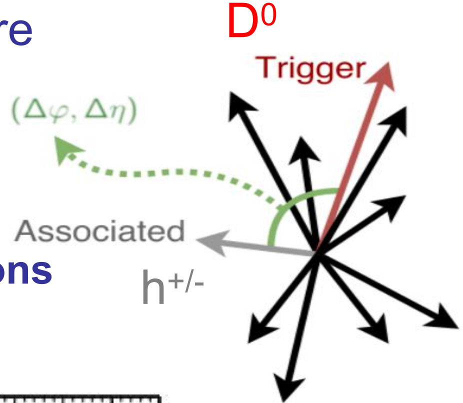
Production & fragmentation: D-h correlations

- **Azimuthal correlation:** jet structure without reconstruction + more
  - Can also be used at low momenta
  - **Near-side** azimuthal correlation: heavy-quark fragmentation
  - **Away-side** peak: recoil jet
  - Further influences: **underlying event, initial- and final-state radiations**

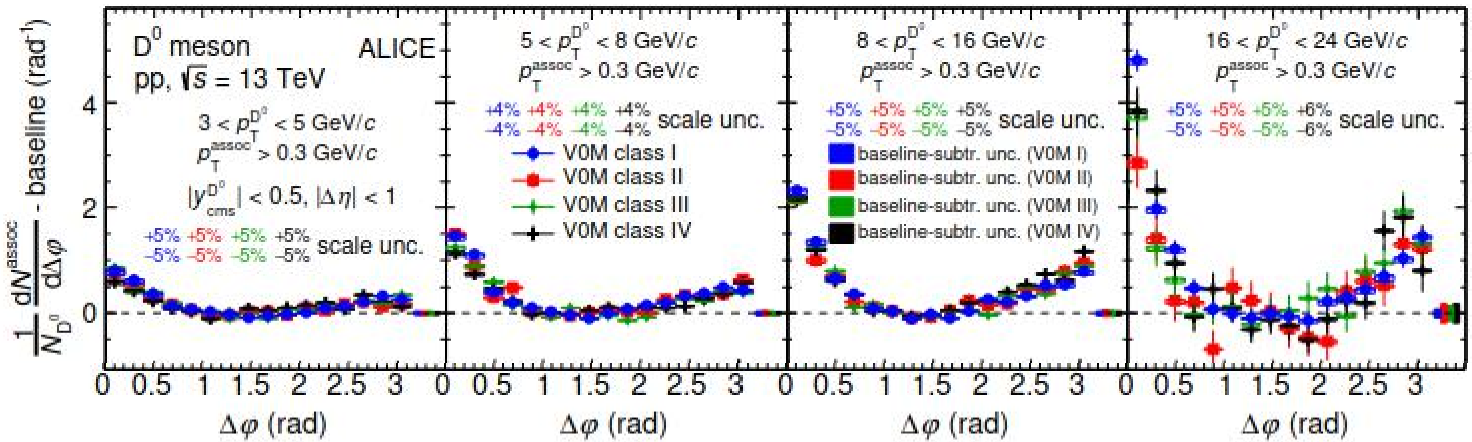


Production & fragmentation: D-h correlations

- **Azimuthal correlation:** jet structure without reconstruction + more
  - Can also be used at low momenta
  - **Near-side** azimuthal correlation: heavy-quark fragmentation
  - **Away-side** peak: recoil jet
  - Further influences: **underlying event, initial- and final-state radiations**



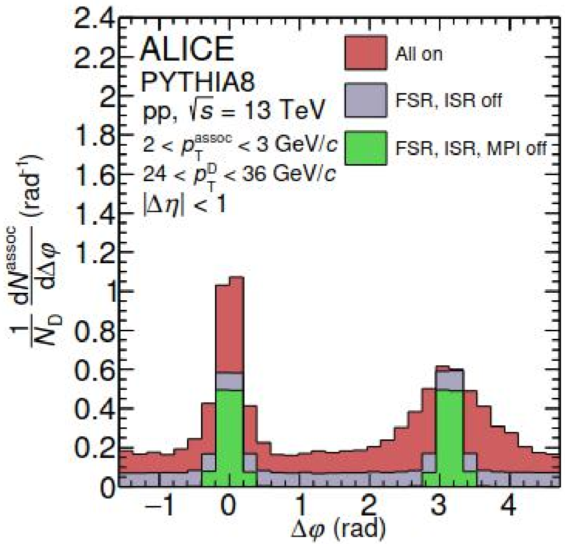
ALICE, EPJ C 82 (2022) 335



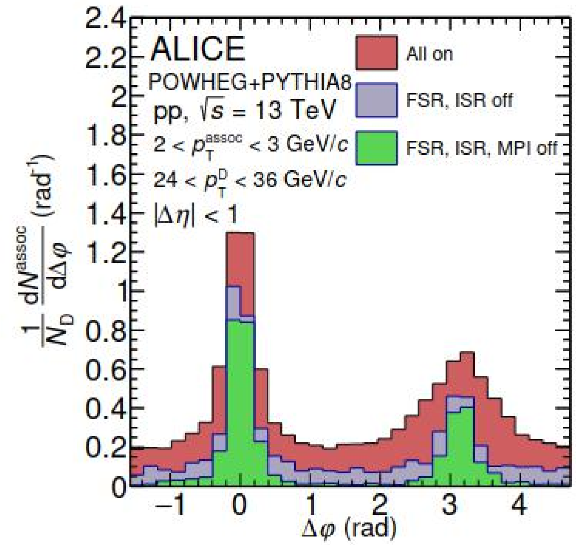
- No significant dependence on the activity of the final state (V0 multiplicy)

Production & fragmentation: D-h correlations

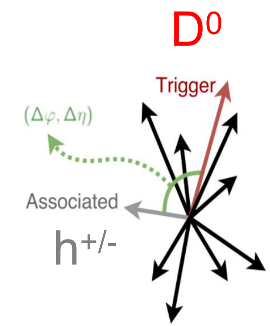
ALICE (incl. E.Frajna), EPJ C 82 (2022) 335



PYTHIA 8
LO matrix element + ISR + FSR



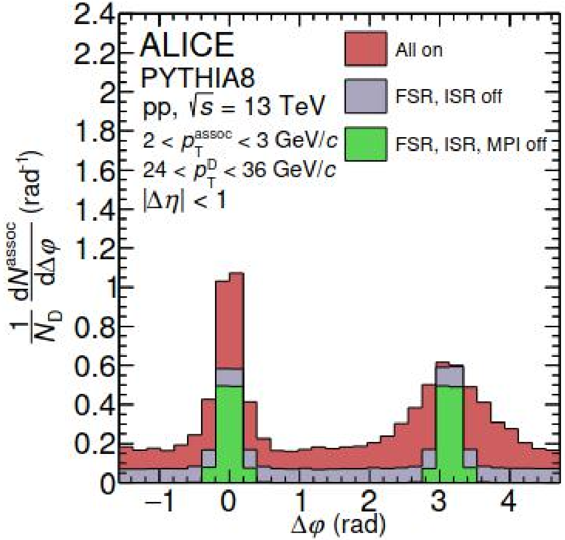
POWHEG+PYTHIA8
NLO matrix element+ ISR + FSR



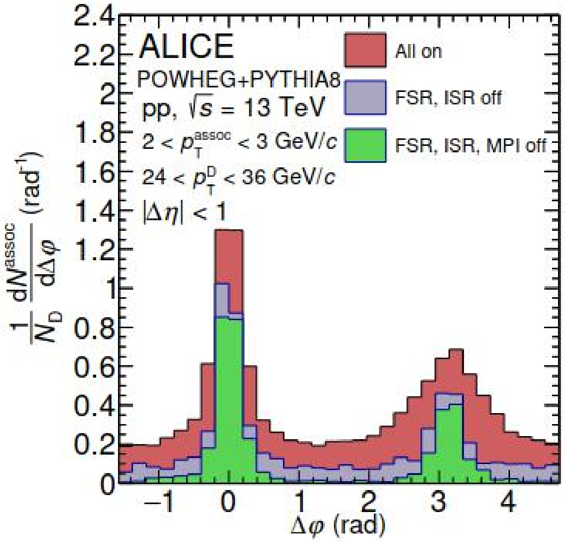
- Separate parton level processes with simulations (ISR, FSR, MPI)

Production & fragmentation: D-h correlations

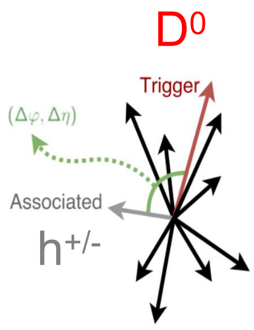
ALICE (incl. E.Frajna), EPJ C 82 (2022) 335



PYTHIA 8
LO matrix element + ISR + FSR



POWHEG+PYTHIA8
NLO matrix element+ ISR + FSR



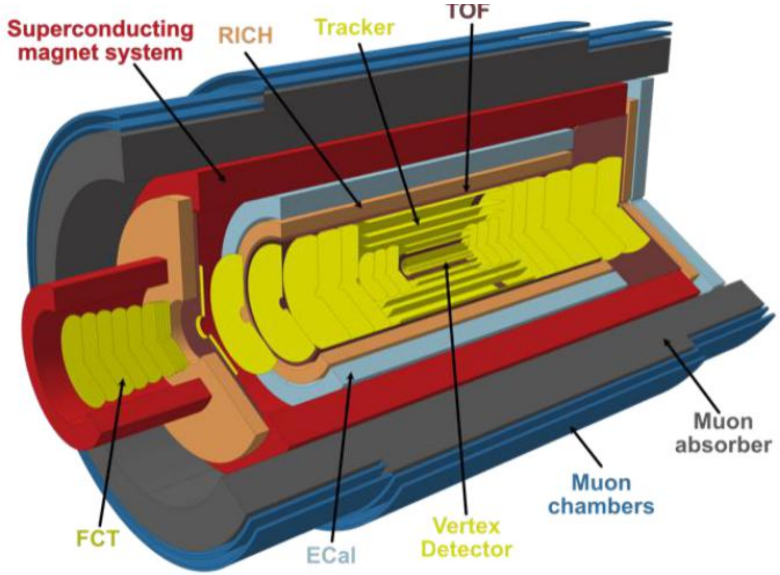
- Separate parton level processes with simulations (ISR, FSR, MPI)
- **Peaks:** more contribution from **initial hard process** in NLO, more **FSR + ISR** in LO
- **Underlying event**: Important role of **multiparton-interactions**

Charm-charm correlations: ALICE3

ALICE 3: the detector of the future

D. Adamová et al., 1902.01211

- Compact multi-purpose detector
- “Ultra-thin” semiconductor technology
- 20-50x higher luminosity

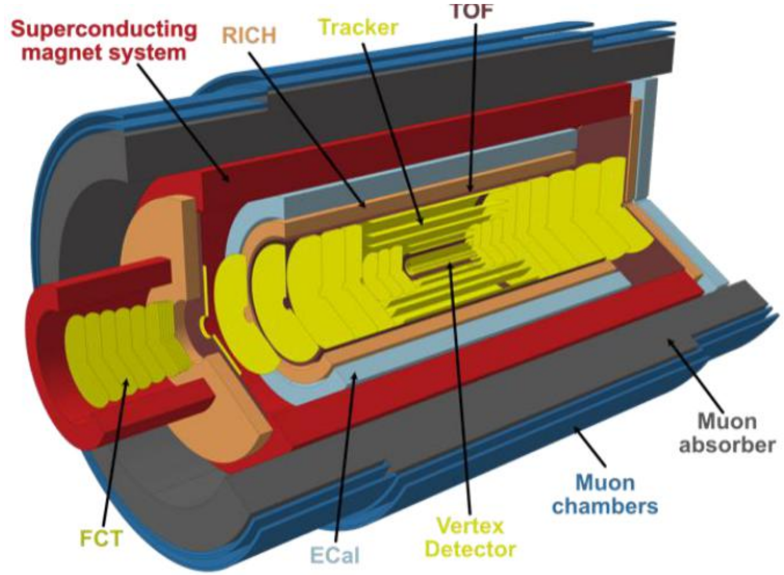


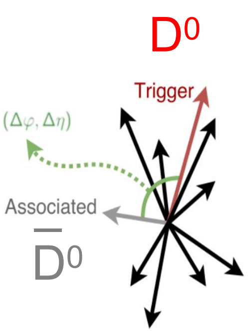
Charm-charm correlations: ALICE3

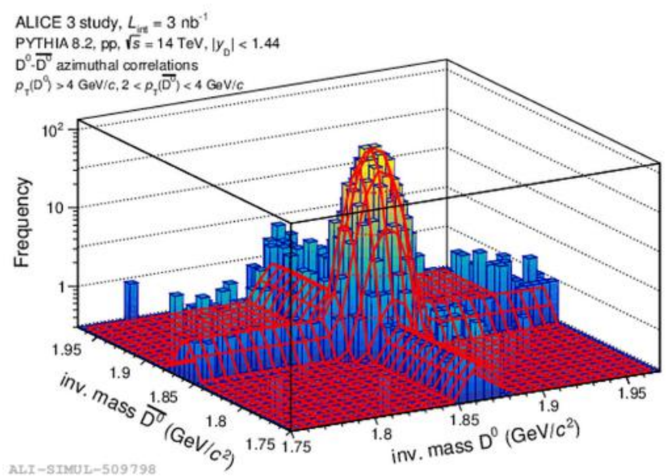
ALICE 3: the detector of the future

D. Adamová et al., 1902.01211

- Compact multi-purpose detector
- “Ultra-thin” semiconductor technology
- 20-50x higher luminosity





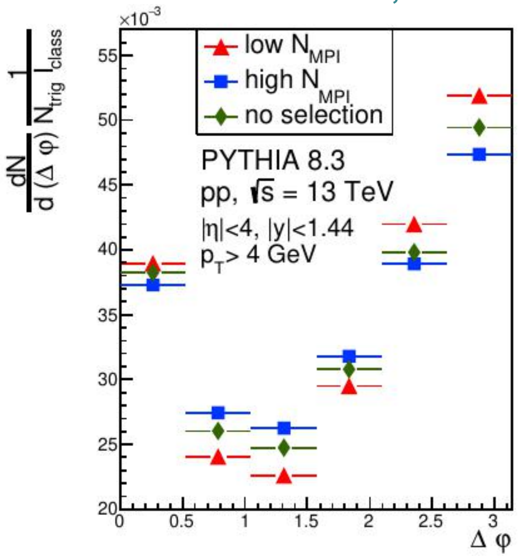


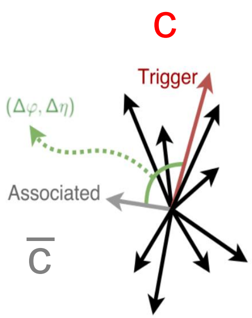
E.Frajna (ALICE), Quark Matter 2022

- Measurements of D^0-$\overline{D}^0$ correlations
- **Separation of individual pQCD processes** (pair production, gluon splitting, flavor excitation) will be possible
- Simulation studies underway

c-c̄ correlations vs. event activity - LO

A Horváth, TDK thesis 2022



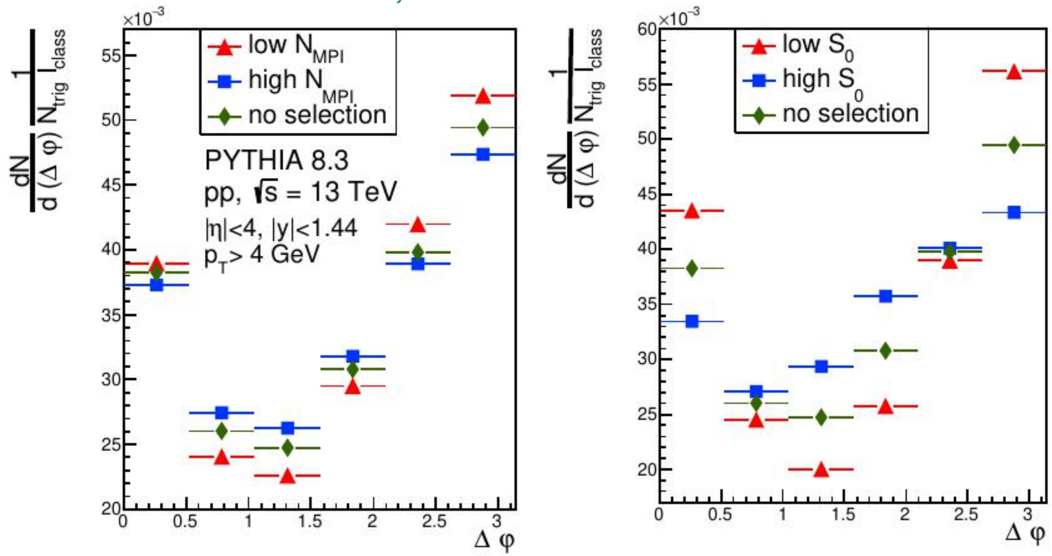


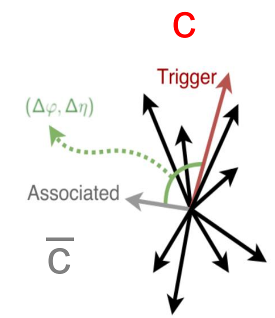
Calculations with PYTHIA 8 (LO)

- **MPI:** more random correlations in high-MPI events (strong UE activity)

c-c̄ correlations vs. event activity - LO

A Horváth, TDK thesis 2022



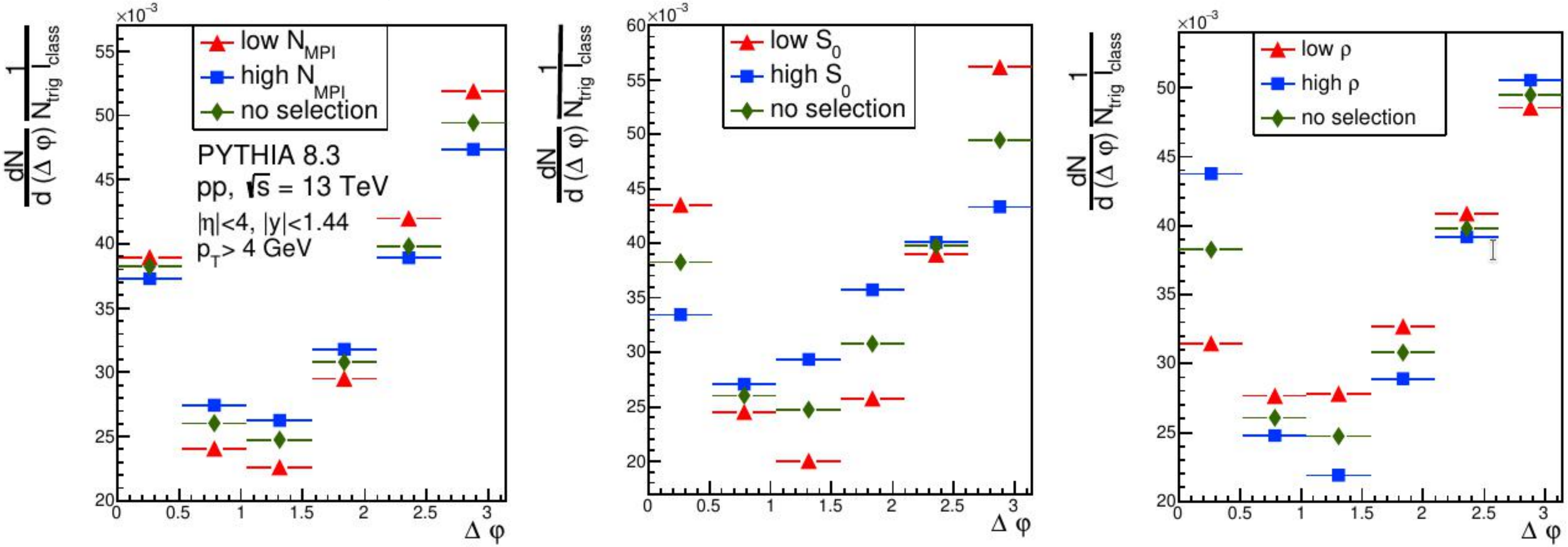


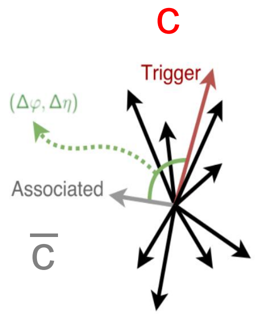
Calculations with PYTHIA 8 (LO)

- **MPI:** more random correlations in high-MPI events (strong UE activity)
- **Spherocity**: more near- and away side correlations in jetty then spherical events

c-c̄ correlations vs. event activity - LO

A Horváth, TDK thesis 2022



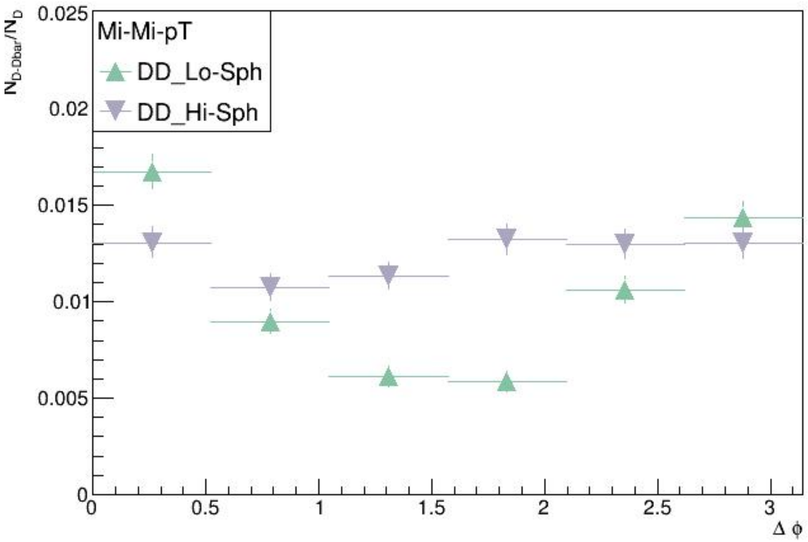


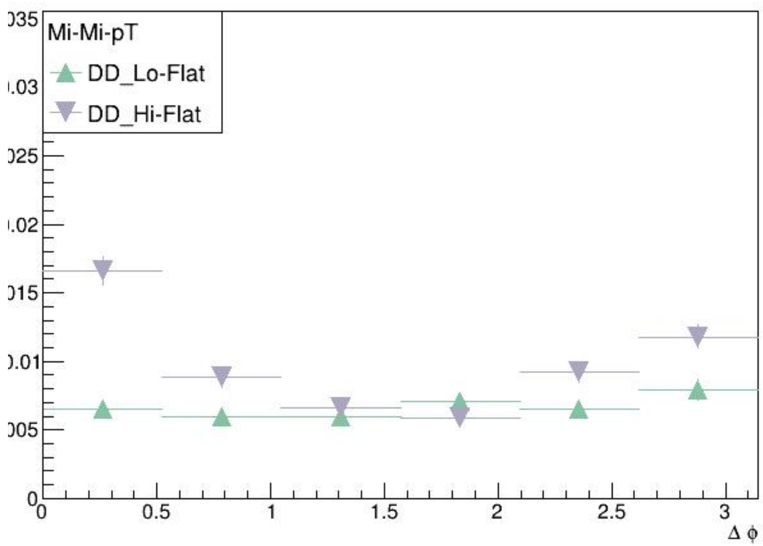
Calculations with PYTHIA 8 (LO)

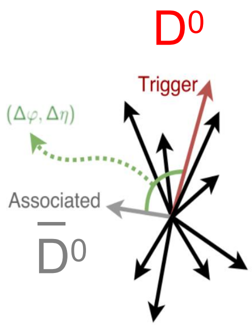
- **MPI:** more random correlations in high-MPI events (strong UE activity)
- **Spherocity**: more near- and away side correlations in jetty then spherical events
- **Flattenicity**: - strong separation at near-side
  - events with gluon-splitting likely tend to be flatter

D-$\bar{D}$ correlations vs. event activity - NLO

E.Frajna, AEPSHEP 2022 poster, shorturl.at/r2489





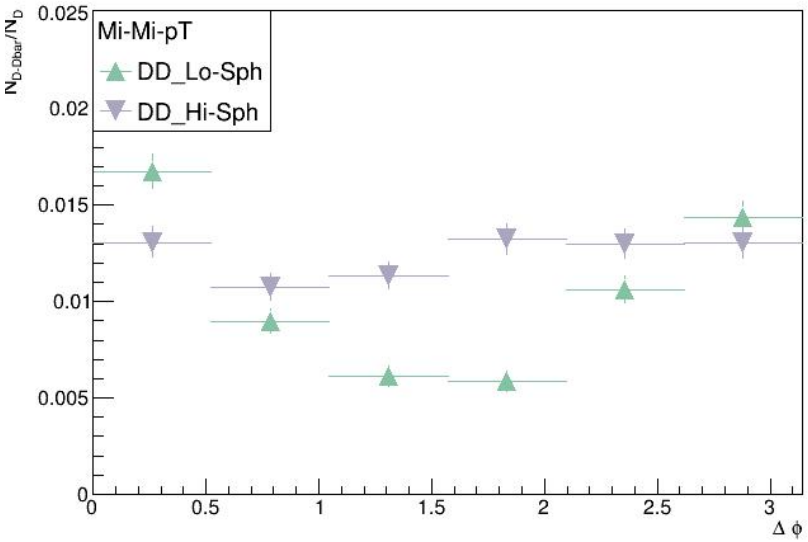


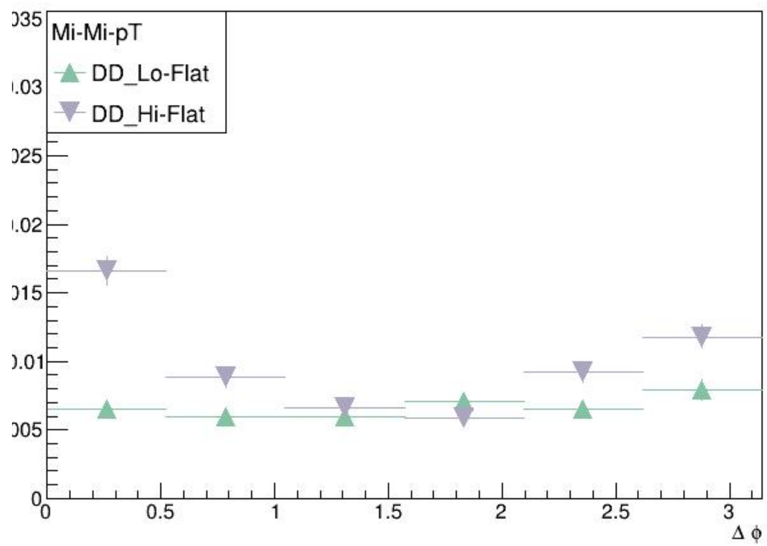
Calculations with HERWIG (Min.bias NLO)

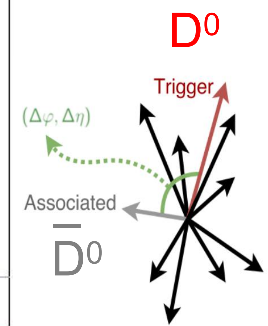
- **Spherocity**: more near- and away side correlations in jetty then spherical events
- **Flattenicity**: - strong separation at near-side
  - events with gluon-splitting likely tend to be flatter

D-D̄ correlations vs. event activity - NLO

E.Frajna, AEPSHEP 2022 poster, shorturl.at/r2489







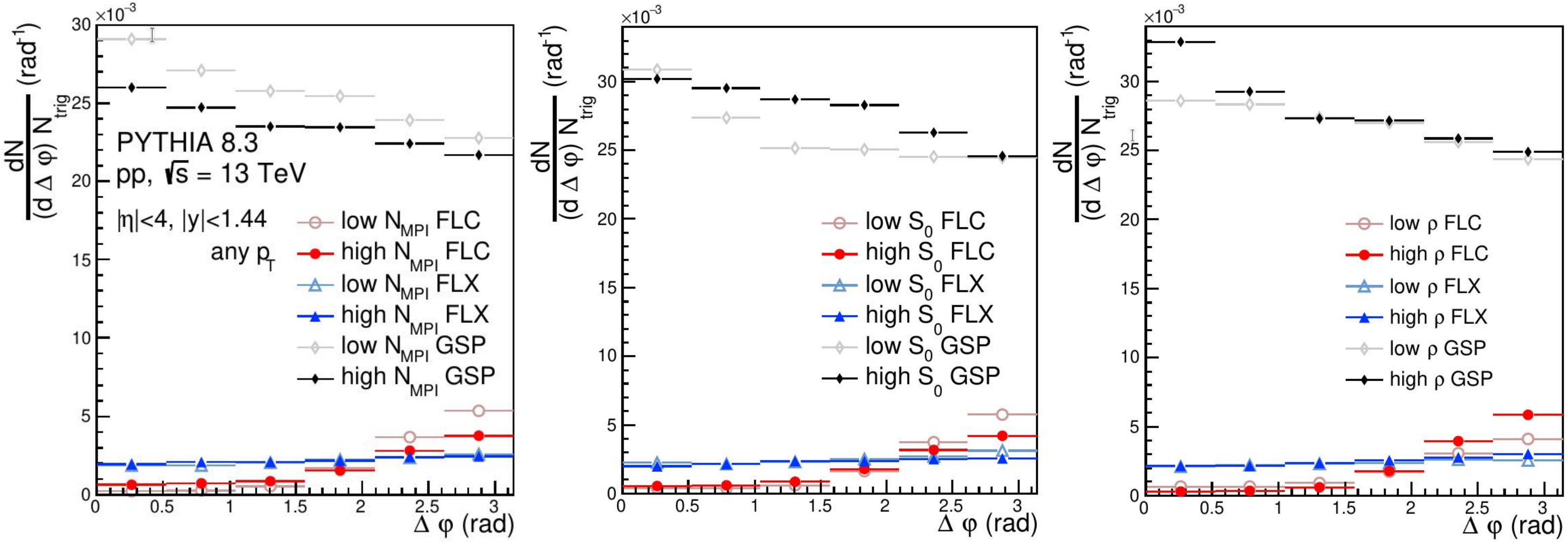
Calculations with HERWIG (Min.bias NLO)

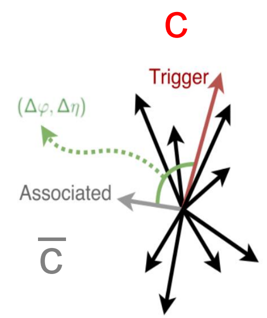
- **Spherocity**: more near- and away side correlations in jetty then spherical events
- **Flattenicity**: - strong separation at near-side
  - events with gluon-splitting likely tend to be flatter

Similar observations in NLO calculations (3-prong) as LO (2-prong + radiations)

Origin of correlations: HF pQCD processes

A Horváth, TDK thesis 2022, shorturl.at/cABS7



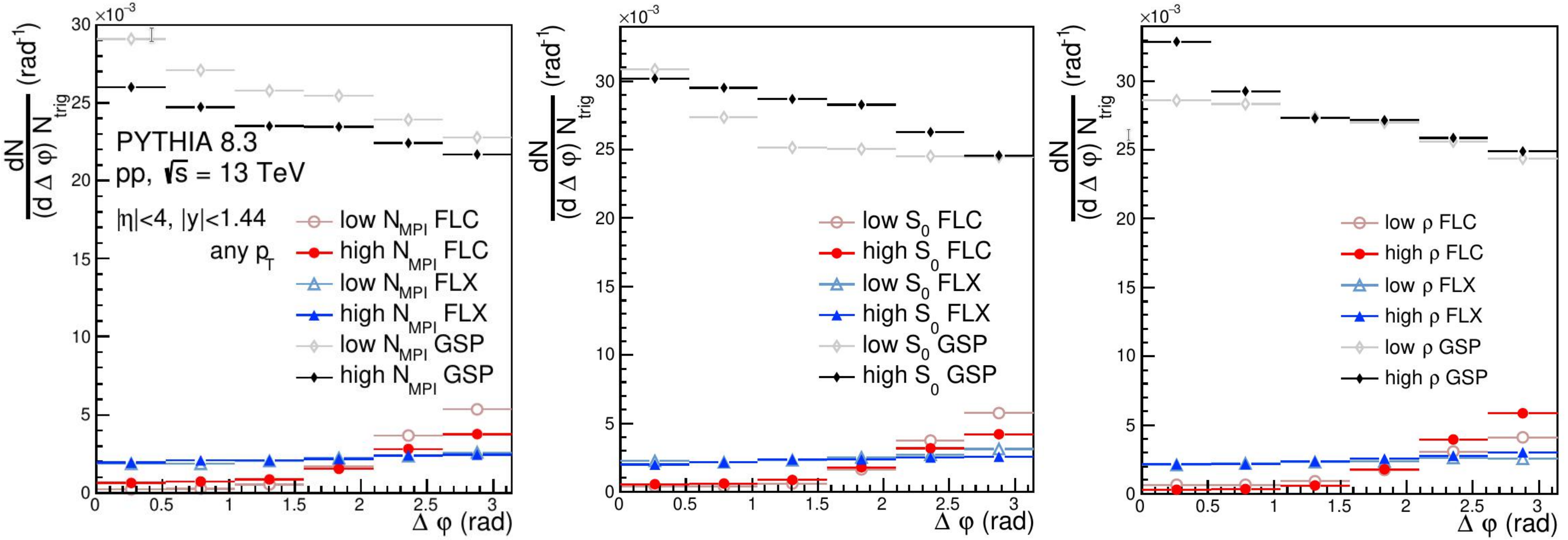


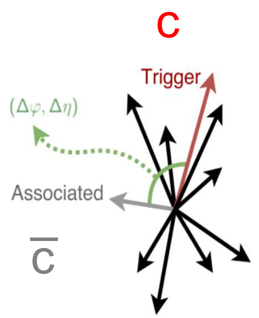
Calculations with PYTHIA 8 (LO)

- **Near-side peak:** mostly produced by gluon splitting
- **Away-side peak:** mostly produced by back-to-back flavor creation

Origin of correlations: HF pQCD processes

A Horváth, TDK thesis 2022, shorturl.at/cABS7





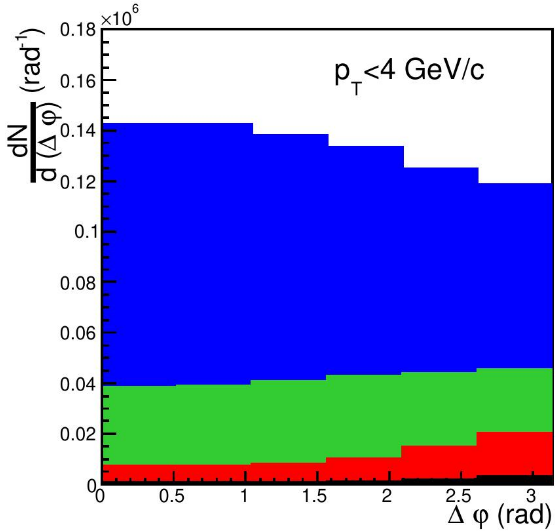
Calculations with PYTHIA 8 (LO)

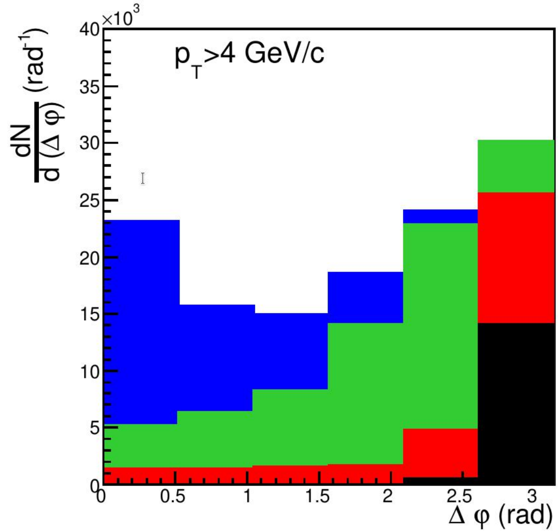
- **Near-side peak:** mostly produced by gluon splitting
- **Away-side peak:** mostly produced by back-to-back flavor creation
- Low flattenicity enhances both peaks, represents high N_{MPI} but more distinctive

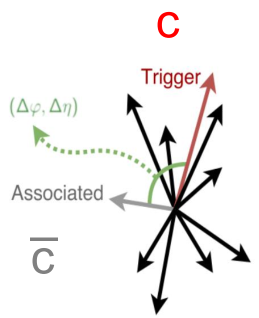
  → **Flattenicity is a great tool to select certain physics processes**

Origin of correlations: parton-level processes

A Horváth, TDK thesis 2022, shorturl.at/cABS7





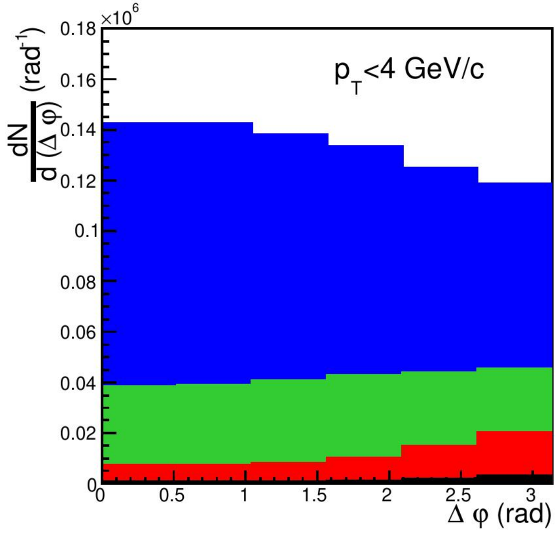


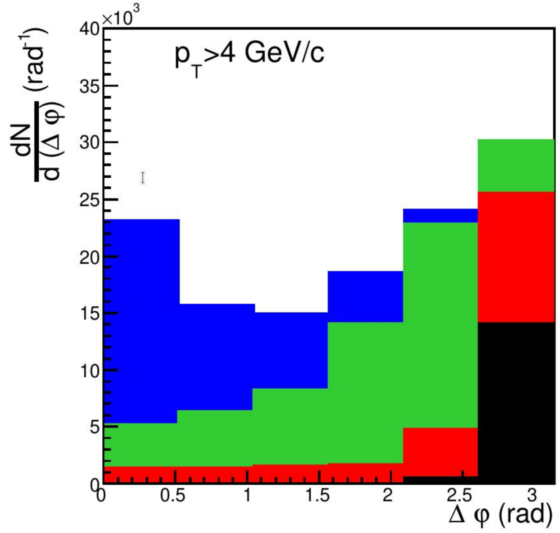
Calculations with PYTHIA 8 (LO)

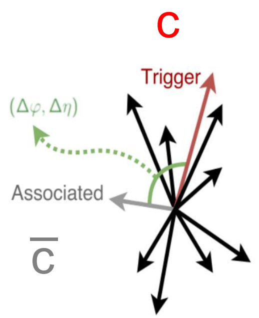
- Contributions strongly depend on p_T
- Away side: Hard process, ISR, MPI
- Near-side: mostly by FSR

Origin of correlations: parton-level processes

A Horváth, TDK thesis 2022, shorturl.at/cABS7





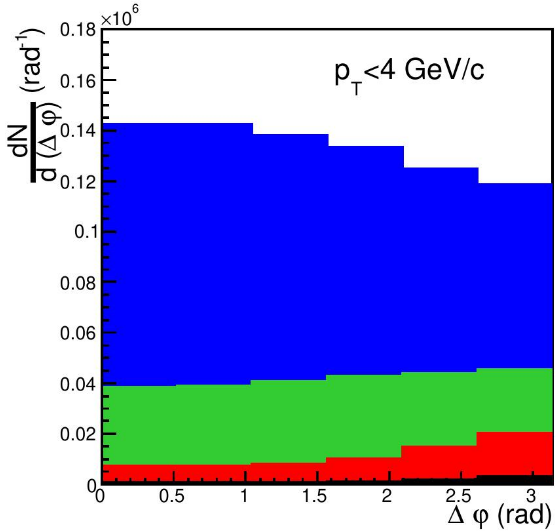


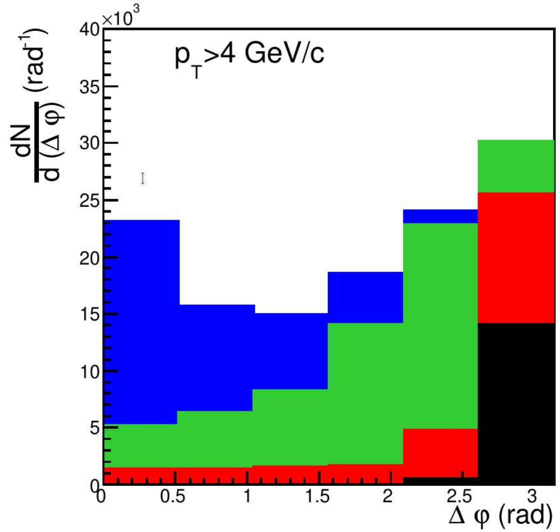
Calculations with PYTHIA 8 (LO)

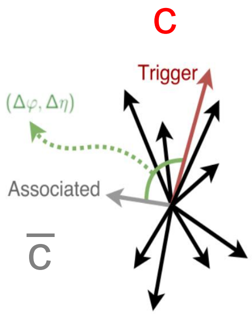
- Contributions strongly depend on p_T
- Away side: Hard process, ISR, MPI
- Near-side: mostly by FSR
- **Can we select partonic processes based on event shape?**

Origin of correlations: parton-level processes

A Horváth, TDK thesis 2022, shorturl.at/cABS7







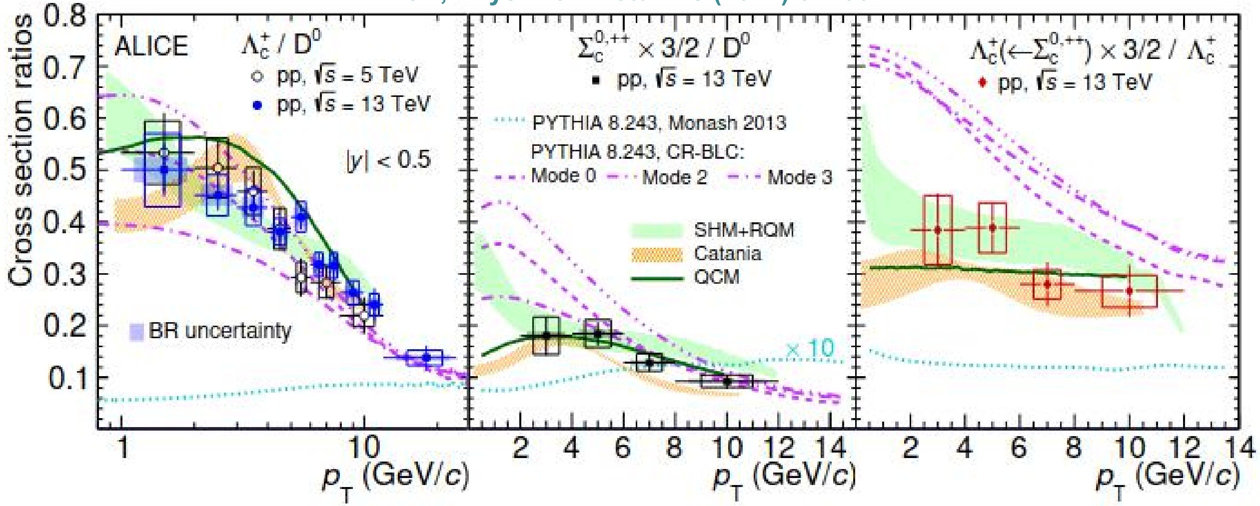
Calculations with PYTHIA 8 (LO)

- Contributions strongly depend on p_T
- Away side: Hard process, ISR, MPI
- Near-side: mostly by FSR
- **Can we select partonic processes based on event shape?**

Anikó Horváth
Event-shape-dependent cc analysis
FLASH TALK: Tuesday 14:15
POSTER: 16:00

Charm baryon enhancement

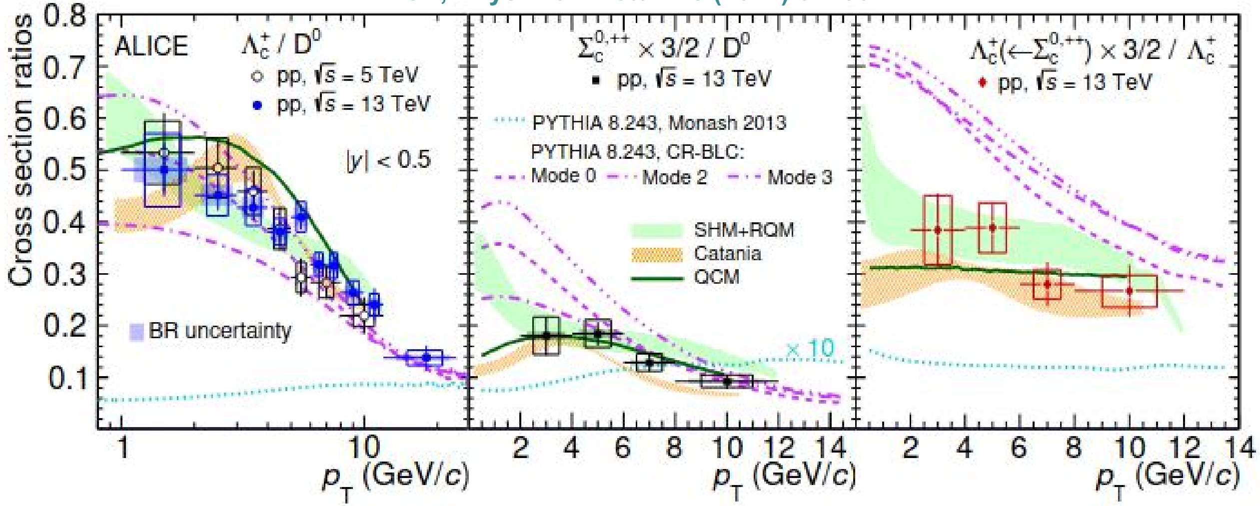
ALICE, Phys. Rev. Lett. 128 (2022) 012001



- **Charm baryon to meson ratios: sensitive probes of fragmentation**

Charm baryon enhancement

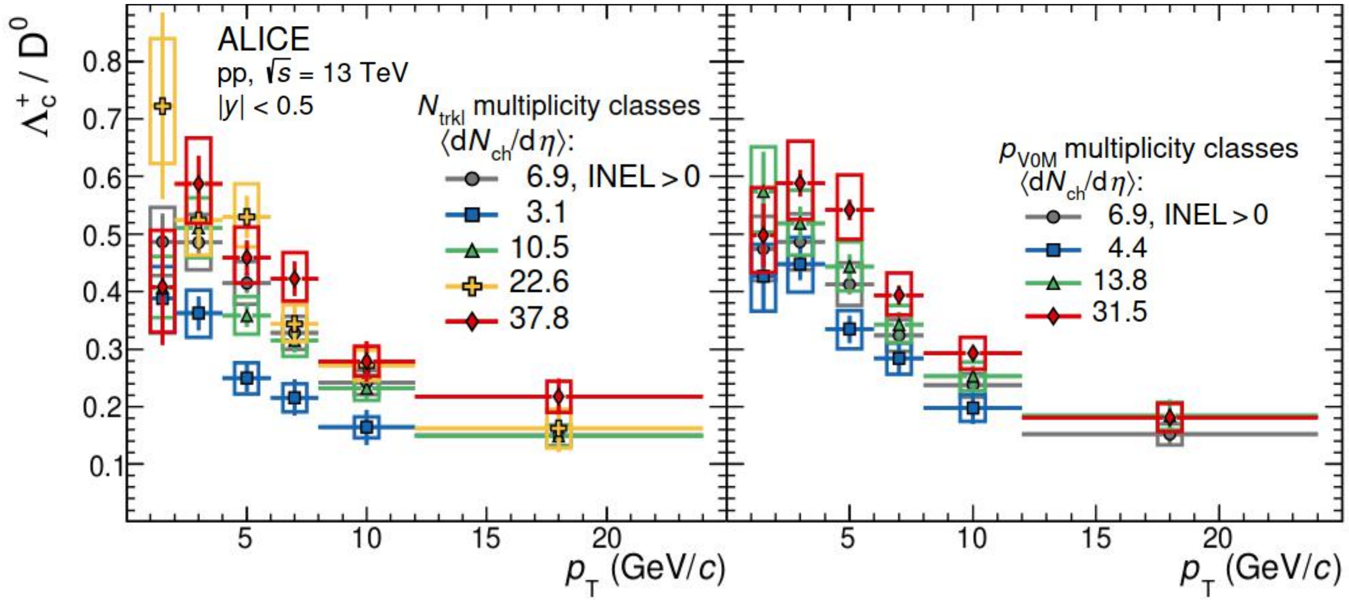
ALICE, Phys. Rev. Lett. 128 (2022) 012001



- **Charm baryon to meson ratios: sensitive probes of fragmentation**
- Λ_c/D^0 and Σ_c/D^0 underestimated by models based on factorization approach with fragmentation functions from ee collisions: **HF fragmentation universality broken!**
  - **PYTHIA 8 CR-BLC**: string formation beyond leading color approximation
  - **Catania**: fragmentation + coalescence of charm and light quarks
  - **SH model + RQM:** feed-down from augmented set of charm-baryon states
  - **QCM:** coalescence model based on statistical weights + “equal quark-velocity

Charm baryon enhancement vs. multiplicity

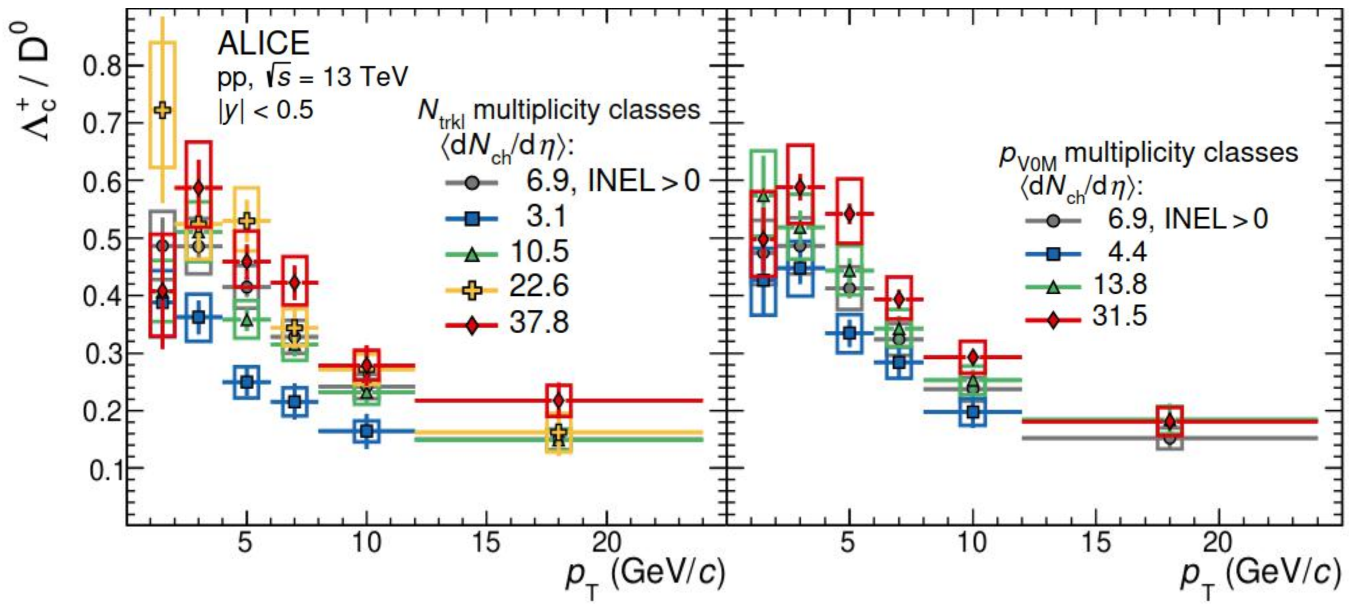
ALICE, PLB 829 (2022) 137065



- The enhancement in Λ_c/D^0 depends on the final state multiplicity at mid- and forward rapidity.
- **Goal: Understand the origin of the enhancement with detailed event activity studies.**

Charm baryon enhancement vs. multiplicity

ALICE, PLB 829 (2022) 137065



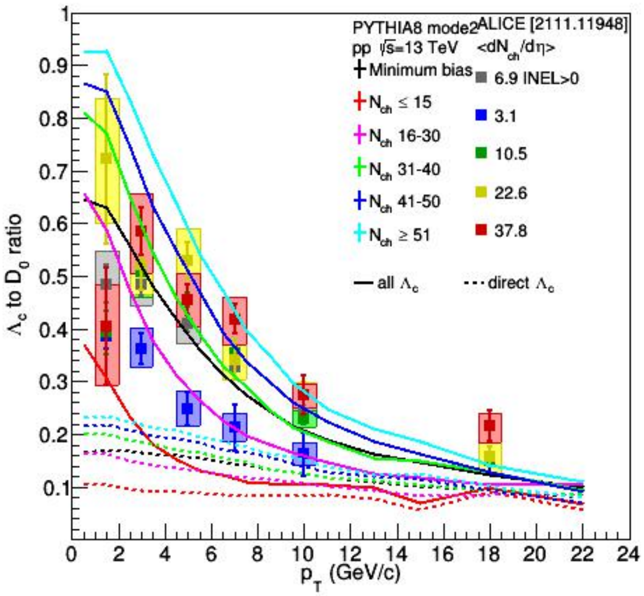
- The enhancement in Λ_c/D^0 depends on the final state multiplicity at mid- and forward rapidity.
- **Goal: Understand the origin of the enhancement with detailed event activity studies.**

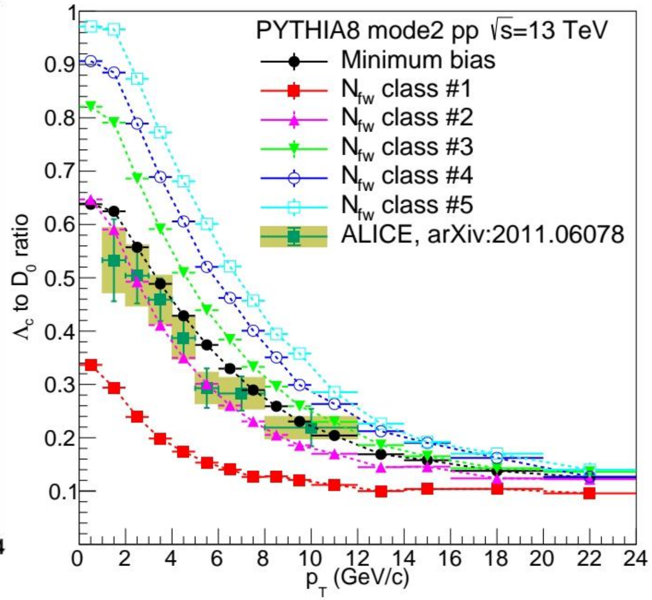
- **Does it come from the jet or the underlying event?**

→ Test sensitivity using predictions with **PYTHIA 8 with the CR-BLC model.**

Λ_C/D^0 ratios vs. central and forward multiplicity

Z.Varga, R.V., J. Phys. G 49 (2022) 075005





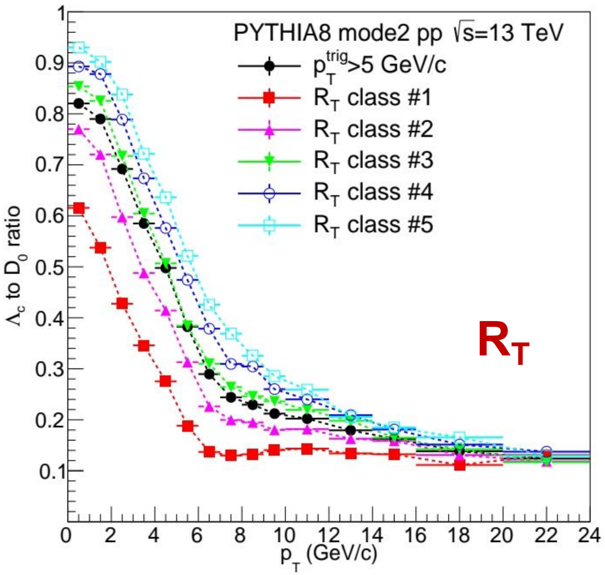
Λ_c(qqc), I = 0

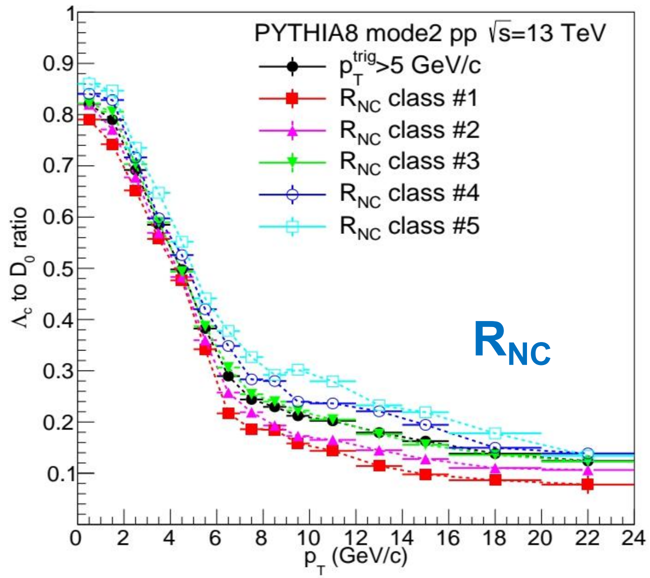
class	#1	#2	#3	#4	#5
N_{ch}	≤15	16–30	31–40	41–50	≥51
N_{fw}	≤45	46–90	91–120	121–150	≥151
R_T	<0.5	0.5–1	1–1.5	1.5–2	>2
R_{NC}	<0.5	0.5–1	1–1.5	1.5–2	>2
S_0	0–0.25	0.25–0.45	0.45–0.55	0.55–0.75	0.75–1

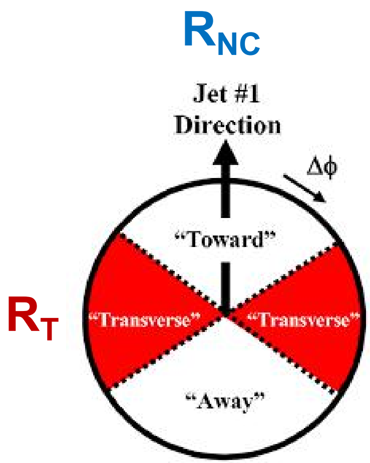
- Simulation results **in agreement with the ALICE** experimental data
- For N_{fw}: a rapidity gap reduces correlation between leading hard processes and multiplicity
- Multiplicity dependence not driven by charm production in jets.
- Recently observed **multiplicity trends** also **predicted**
- p_T dependence may be sensitive to baryon type

Λ_C/D^0 ratios vs. UE and jet activity (triggered)

Z.Varga, R.V., J. Phys. G 49 (2022) 075005



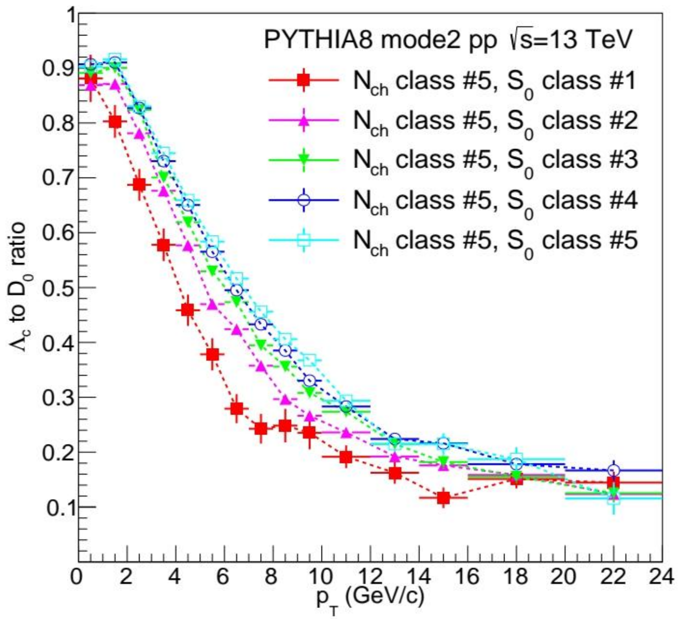




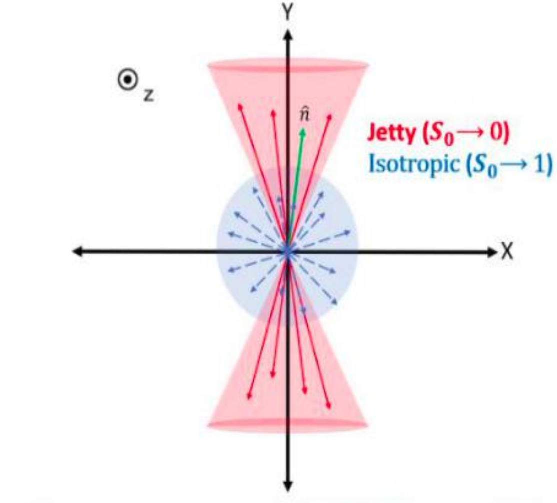
- Events require p_T>5 GeV/c **hadron trigger**.
- Significant difference is observable in case of **R_T (UE activity)**.
- No significant difference when classified by **R_{NC}** classes **(jet activity)**.

Λ_C/D^0 ratios vs. spherocity (minimum bias)

Z.Varga, R.V., J. Phys. G 49 (2022) 075005

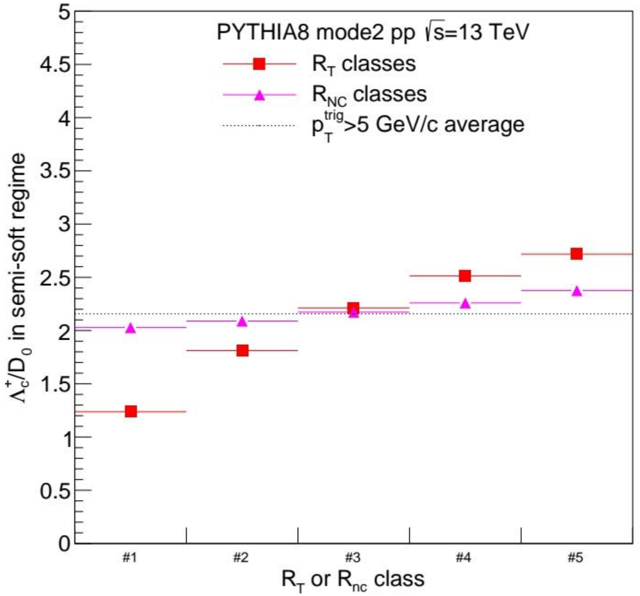


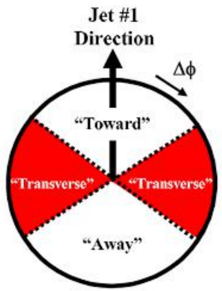
$N_{CH} > 50$

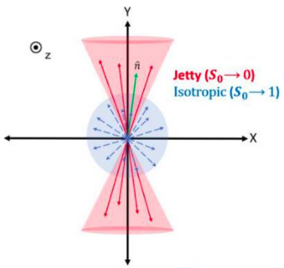


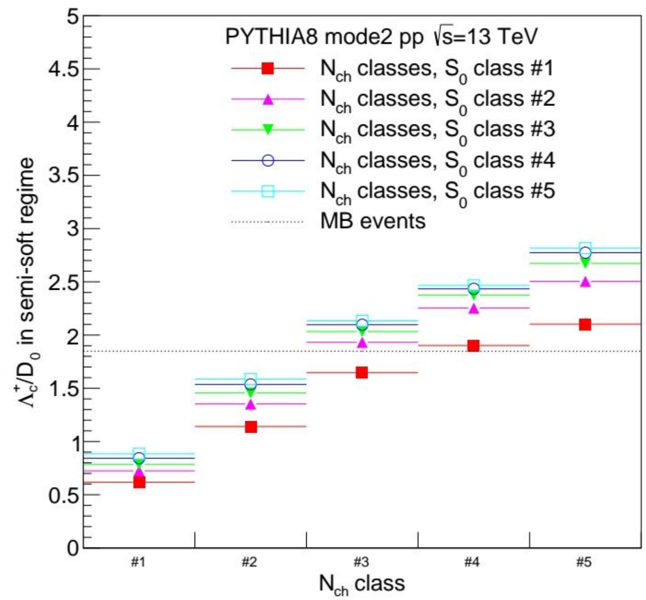
- Spherocity provides a measure for the **jettines**/**isotropy** of events.
- **Significant difference** is observed **for different spherocity classes** at fixed event-multiplicity.

Λ_C/D^0 ratios - triggered vs. minbias









R_T, R_{NC}; $p_T^{trigger} > 5$ GeV/c

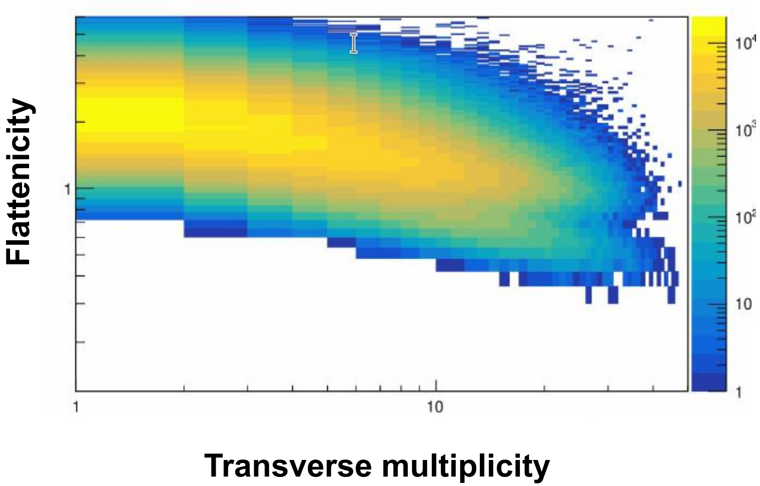
- Hard process required
- **Strong dependence** of ratios on the **UE activity**,
- **No pronounced dependence** on the **jet multiplicity**.
- Trigger biases sample and decreases statistics

S_0: Jettiness in minimum-bias events

- No need to use a trigger
- At high final-state multiplicity: ratio depends on jettiness,
- Low final-state multiplicity: dependence is minute

Λ_C/D^0 ratios - flattenicity?

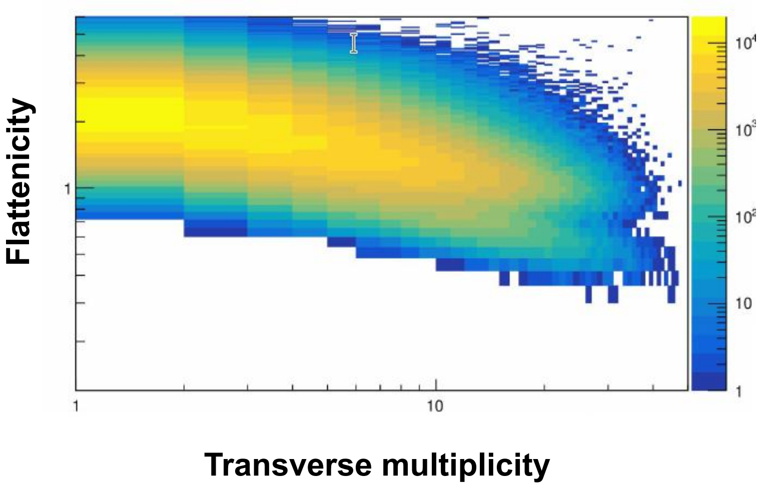
E.Frajna, AEPSHEP 2022 poster, shorturl.at/r2489

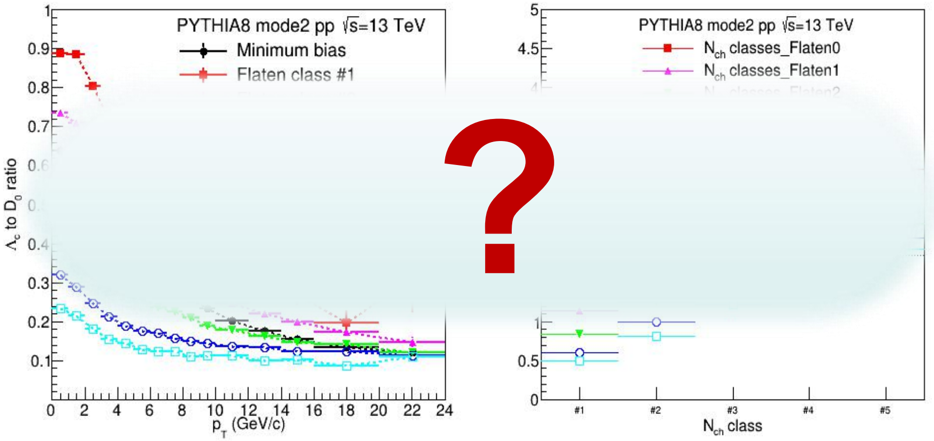


Flattenicity:

- Correlates well with N_{MPI} and N_{trans}
- Works in minimum-bias events
- Proven to be selective for specific heavy-flavor processes
- **Is it useful to address the charmed-baryon enhancement?**

E.Frajna, AEPSHEP 2022 poster, shorturl.at/r2489





Flattenicity:

- Correlates well with N_{MPI} and N_{trans}
- Works in minimum-bias events
- Proven to be selective for specific heavy-flavor processes
- **Is it useful to address the charmed-baryon enhancement?**

Zoltán Varga
Event-shape-dependent cc analysis
FLASH TALK: Thursday 14:18
POSTER: 16:00

Summary

Heavy-flavor probes in small systems

- Collective behavior likely caused by complex but “cold” QCD processes
- Detailed, event-shape dependent probes help in understanding these
- Process-level analysis possible using simulations

Summary

Heavy-flavor probes in small systems

- Collective behavior likely caused by complex but “cold” QCD processes
- Detailed, event-shape dependent probes help in understanding these
- Process-level analysis possible using simulations

Azimuthal correlations of heavy-flavor mesons

- LO and NLO simulations show that none of the event classifiers (eg. spherocity, flatenicity) are perfect representations for the MPI
- These might however provide constraints that select specific QCD processes

Summary

Heavy-flavor probes in small systems

- Collective behavior likely caused by complex but “cold” QCD processes
- Detailed, event-shape dependent probes help in understanding these
- Process-level analysis possible using simulations

Azimuthal correlations of heavy-flavor mesons

- LO and NLO simulations show that none of the event classifiers (eg. spherocity, flatenicity) are perfect representations for the MPI
- These might however provide constraints that select specific QCD processes

Charmed-baryon enhancement

- Several proposed event classifiers are sensitive probes of the Lc enhancement
- PYTHIA8 + CR-BLC predicts that enhancement comes from the underlying event

Summary

Heavy-flavor probes in small systems

- Collective behavior likely caused by complex but "cold" QCD processes
- Detailed, event-shape dependent probes help in understanding these
- Process-level analysis possible using simulations

Azimuthal correlations of heavy-flavor mesons

- LO and NLO simulations show that none of the event classifiers (eg. spherocity, flatenicity) are perfect representations for the MPI
- These might however provide constraints that select specific QCD processes

Charmed-baryon enhancement

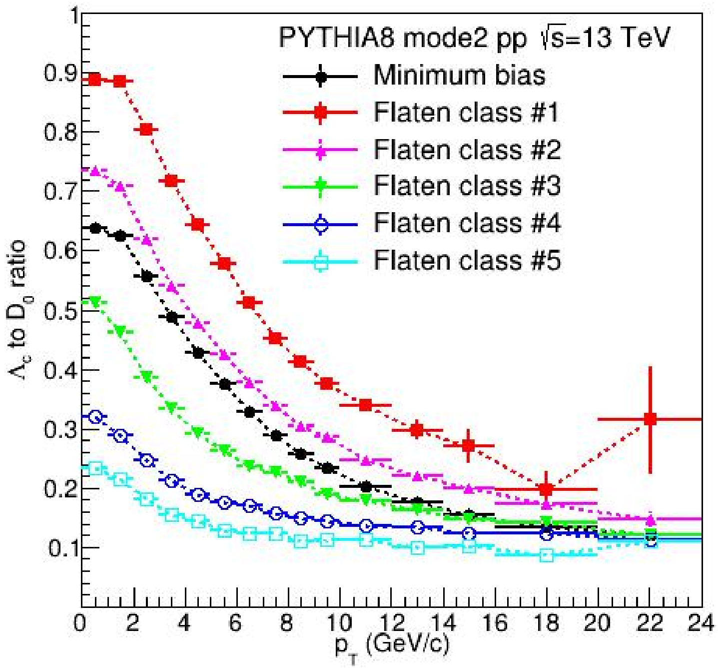
- Several proposed event classifiers are sensitive probes of the Lc enhancement
- PYTHIA8 + CR-BLC predicts that enhancement comes from the underlying event

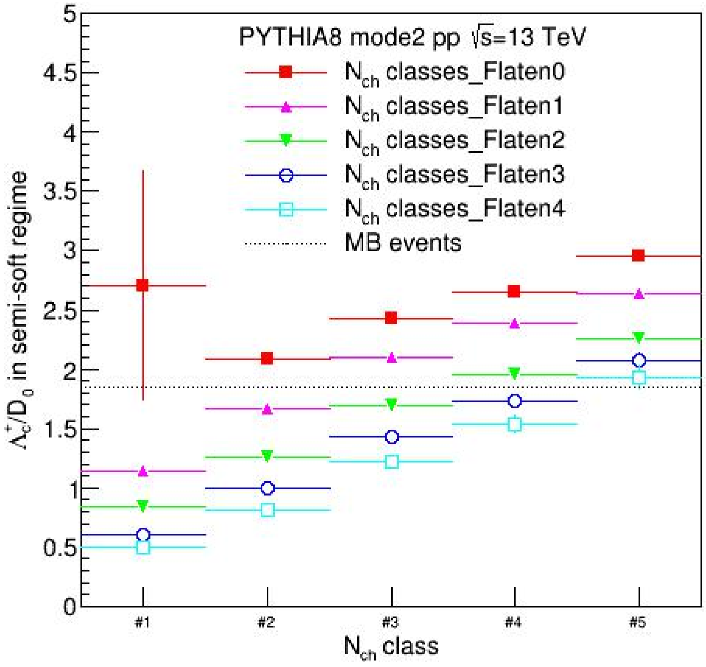
New era with LHC Run3 and future ALICE3 experiment

- Simulations provide predictions and test sensitivity
- Ultimately, predictions can be verified only with detailed experimental data


Thank you!

Λ_C/D^0 yield for different flatenicity classes





- For lower flatenicity, the event is more flat up to high rapidities.
- The $\boldsymbol{\Lambda_C/D^0}$ ratio decreases with increasing flatenicity.
- **Flatenicity correlates with $\mathbf{N_{ch}}$**, might be a better quantity than spherocity to represent MPI.