Hadronization within the Non-Extensive Approaches

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Abstract
We review transverse momentum distributions of various identified charged particles stemming from relativistic heavy ion collisions within the non-extensive approach. In particular investigations on the fitting χ^2/ndf show that mass scaling becomes more explicit with heavier produced charged hadrons in pp as well as heavy-ion collisions. The spectra shape, the temperature T and the Tsallis non-extensive parameter q, do exhibit linear dependence.

Introduction

Recently more and more attention has been paid to the analysis of transverse momentum (p_T) spectra in heavy ion collisions in the non-extensive approach [1]. As a basic quantity measured in experiments, the p_T spectrum reveals useful information on the dynamics of the colliding systems. Rather it has been realized that data within high p_T region on many single-particle distributions show a power-law than an exponential behaviour. This does not expect from the usual statistical models based on Boltzmann-Gibbs (BG) statistics.

Due to the high multiplicities produced in heavy-ion collisions, even in pp, one may use the statistical models to study the mechanism. Clearly, the identified particle spectra at RHIC and LHC do not satisfy the usual BG distribution at high p_T region. In the last thirty years the Tsallis distribution has been frequently used in heavy-ion collisions. It is based on the generalised $q-$exponential function:

$$\exp_q(x) := [1+(1-q)x]^{\frac{1}{1-q}} \tag{1}$$

In this work we review p_T spectra distributions within Tsallis non-extensive approaches, both in pp and nucleus-nucleus collisions. Different p_T distribution formulas are investigated and compared in order to reflect their connections and differences.

Coordinate system and spectra

In high energy physics, one useful variable used commonly to describe the kinematic condition of a particle is the rapidity variable y. It is defined in terms of its energy-momentum components E and p_z (z is the beam axis). [2]

$$y \equiv \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \frac{1}{2}\ln\frac{1+v_z}{1-v_z} \tag{2}$$

giving $v_z = \tanh y$ and $\gamma_z = \frac{1}{\sqrt{1-v_z^2}} = \frac{E}{\sqrt{E^2-p_z^2}} = \frac{E}{m_T} = \cosh y$. Thus, $E = \gamma_z m_T = m_T\cosh y$.

In many experiments it is only possible to measure the angle of the detected particle related to the beam axis. In this case it turns convenient to utilize it by using the pseudo-rapidity η which is given by [3]

$$\eta = \frac{1}{2}\ln\frac{p+p_z}{p-p_z} = \ln\sqrt{\frac{1+\cos\theta_z}{1-\cos\theta_z}} = \ln[\cot\frac{\theta_z}{2}]. \tag{3}$$

One easily claims their connection as being

$$\frac{dy}{d\eta} = \frac{1}{2}\frac{d}{d\eta}\ln\frac{E+p_z}{E-p_z} = \frac{p}{E} \tag{4}$$

leading to $dN/dy = \frac{E}{p}dN/d\eta$.

In high energy physics, one investigates the Lorentz-invariant particle spectrum $E\frac{dN}{d^3p}$. Fix p_T (or m_T), $dp_z = m_T\cosh y dy = Edy \Rightarrow \frac{dp_z}{E} = dy$ resulting in

$$E\frac{dN}{d^3p} = \frac{dN}{dyd^2p_T} = \frac{dN}{dyp_Tdp_Td\phi_p} = \frac{dN}{dym_Tdm_Td\phi_p} \tag{5}$$

and

$$\frac{dN}{d\eta d^2p_T} = \sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\frac{dN}{dyd^2p_T}, \tag{6}$$

when $\eta = y = 0$, at mid-rapidity we obtain $\frac{dN}{d\eta d^2p_T}|_{\eta=0} = \frac{p}{E}\frac{dN}{dyd^2p_T}|_{y=0} = \frac{p_T}{m_T}\frac{dN}{dyd^2p_T}|_{y=0}$.

Identified hadron spectra

In order to describe hadron spectra stemming from various heavy ion collisions, one has to disentangle effects of a possible transverse flow on spectra and test whether the result complies with the thermal assumption; i.e. that the dependence on momenta is through a dependence on the kinetic energy, $E-\mu \approx E-m$ only. [4]

The analysis is due to the fact that the source emitting the detected hadrons is flowing in all directions. Here the 4-velocity of the source and the actual 4-momentum of the particle are parameterized by rapidity and coordinate-rapidity:

$$\begin{aligned} u_\mu &= (\gamma_T\cosh y', \gamma_T\sinh y', \gamma_T v_T\cos\phi, \gamma_T v_T\sin\phi) \\ p_\mu &= (m_T\cosh y, m_T\sinh y, p_T\cos\psi, p_T\sin\psi) \end{aligned} \tag{7}$$

leading to

$$E = u_\mu p^\mu = \gamma_T m_T\cosh(y-y') - \gamma_T v_T p_T\cos(\psi-\phi) \tag{8}$$

Various approximations

- More and more experiments present the high-energy multi-particle production spectra as a power-law distribution, well described by the formula

$$E\frac{d^3N}{dp^3} \propto f_q(-\frac{E}{T}) \tag{9}$$

where $f_q(x) = \exp_q(x)$ of Eq.(1) is a generalized distribution function of the BG one (which is a special case of $q=1$).

- Others go further to determine the normalization constant which also depends on the parameter m (mass of particle), T (fitting temperature), q (or **n=1/(q-1)**) the non-extensive parameter)
- Moreover, from the total number of particles $N = gV\int\frac{d^3p}{(2\pi)^3}f_q(-\frac{E}{T})$ we can exhibit the spectra as

$$E\frac{d^3N}{dp^3} = \frac{gV}{(2\pi)^3}Ef_q(-\frac{E}{T}) \tag{10}$$

For all the above [5], with respect to the relation of Eq.(8) the limitation corresponds to

1. $v_T \to 0$, then $E \to m_T\cosh(y-\eta)$.
2. $v_T \to 0$ and $y - y' \sim 0$ (midrapidity), then $E \to m_T$ or $E = m_T - m$ with $\mu \sim m$.

Next we focus on some of the formulas used for approximating the identified particle spectra in various collisions.

Fitting functions of spectra $\frac{1}{N}\frac{1}{2\pi p_T}\frac{d^2N}{dydp_T}|_{|y|=0} =$

scaling	simple	thermal consistent	normalized
m_T-m	$A_1\cdot(1+\frac{m_T-m}{nT})^{-n}$	$A_2\cdot m_T(1+\frac{m_T-m}{nT})^{-n}$	f_3
m_T	$A_5\cdot(1+\frac{m_T}{nT})^{-n}$	$A_4\cdot m_T(1+\frac{m_T}{nT})^{-n}$	
p_T	$A_6\cdot(1+\frac{p_T}{nT})^{-n}$		

where $f_3 = A_3\cdot\frac{1}{2\pi T(T+m)}(1+\frac{m_T-m}{nT})^{-n}$. In this work we compare the differences of m_T-m and m_T scalings as well as the simple p_T scaling with full space region:

$$\int_{-\pi}^{+\pi}E\frac{dN}{d^3p} = \int_{-\pi}^{+\pi}\frac{dN}{dyp_Tdp_Td\phi_p} = \frac{1}{2\pi p_T}\frac{d^2N}{dydp_T} \tag{11}$$

Results and discussions

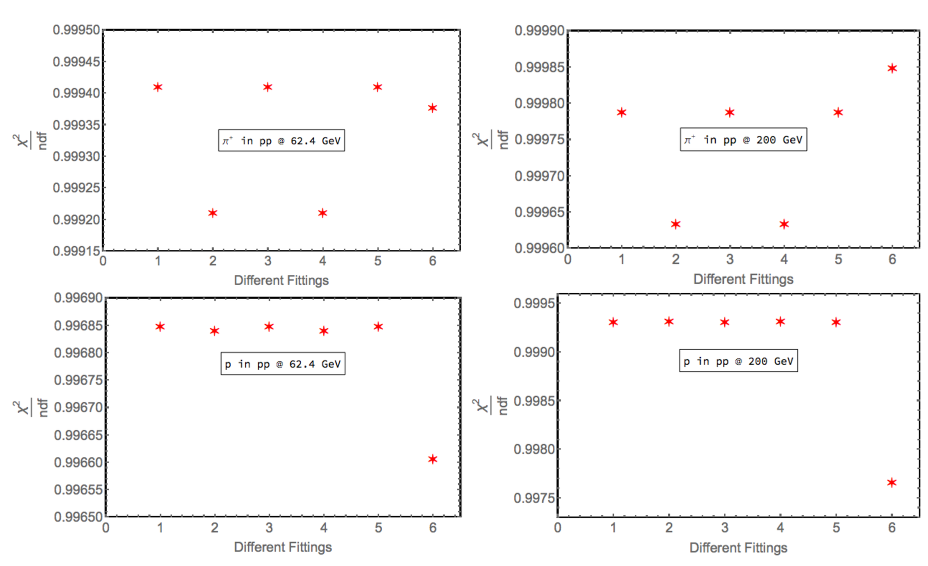

Figure 1: χ^2/ndf in pp collisions for different m of identified particles.

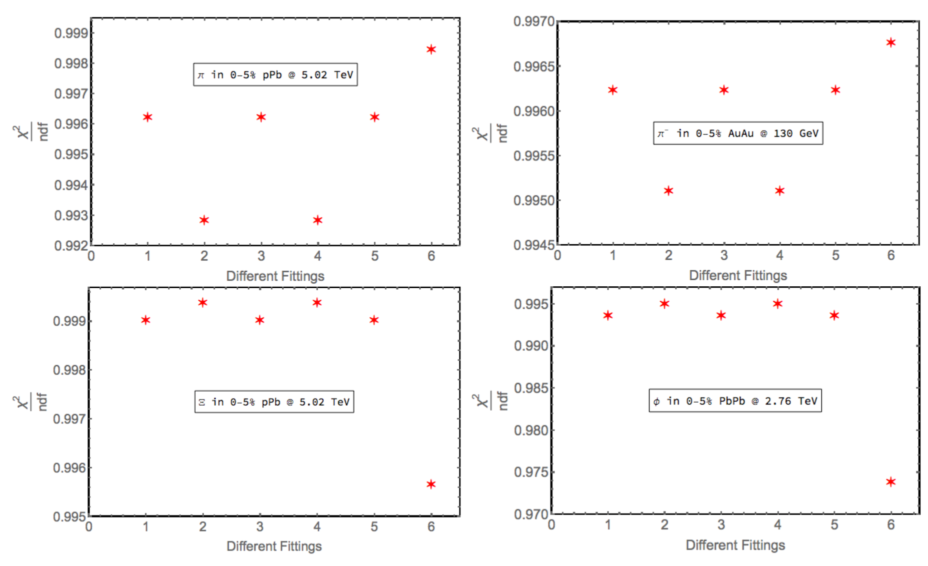

Figure 2: χ^2/ndf in heavy ion collisions for different m.

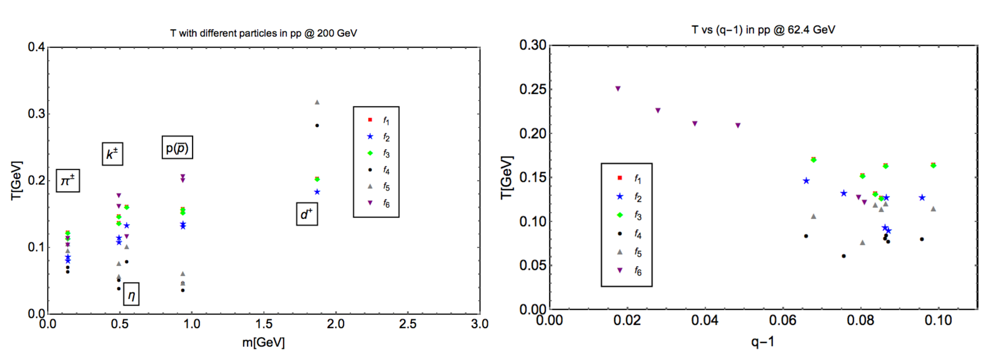

Figure 3: parameter analysis from pp

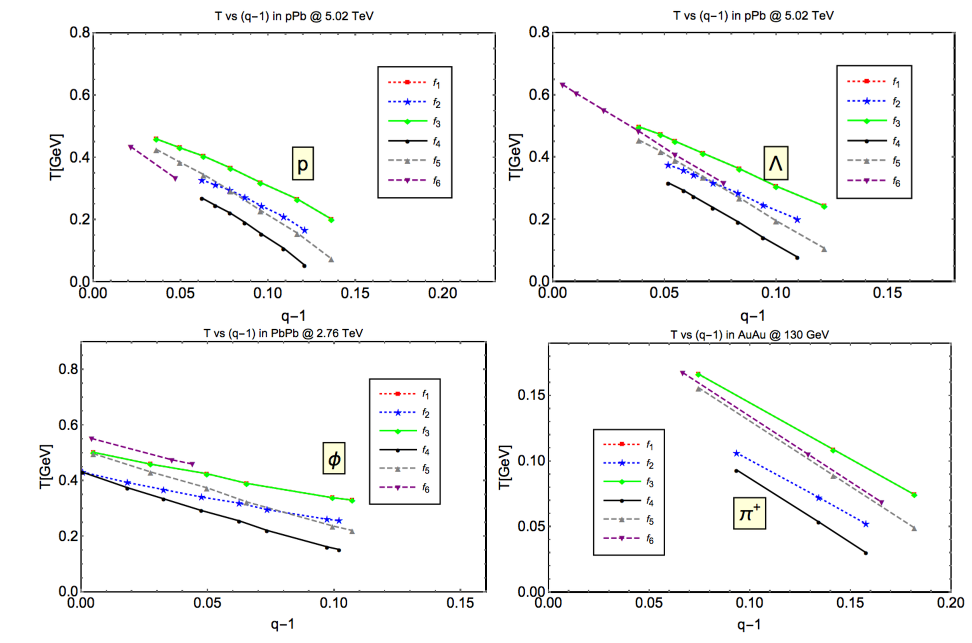

Figure 4: correlations between T and $q-1 = 1/n$ for different identified particle spectra in heavy ion collisions

In all figures, the six different spectra-fitting formulas correspond to the six functions in the table above, namely, $f_1 = A_1\cdot(1+\frac{m_T-m}{nT})^{-n}$, $f_2 = A_2\cdot m_T(1+\frac{m_T-m}{nT})^{-n}$, $f_3 = A_3\cdot\frac{1}{2\pi T(T+m)}(1+\frac{m_T-m}{nT})^{-n}$, $f_4 = A_4\cdot m_T(1+\frac{m_T}{nT})^{-n}$, $f_5 = A_5\cdot(1+\frac{m_T}{nT})^{-n}$ and $f_6 = A_6\cdot(1+\frac{p_T}{nT})^{-n}$ for different approximations with respect to the mass of produced charged particles in collisions. Data are taken from pp[6], pA[7] and AA[8] collisions in RHIC and LHC experimental groups. And we apply all fitting functions on the data within as wide p_T range as possbile but mid-rapidity region $|y| < 0.5$.

Conclusions

- The first two figures tell us that f_1, f_3 and f_5 seem to have the same χ^2, which means no big differences just for fittings. Checking their fitting parameters (A, T and n/q) we observe that f_1 and f_3 share the same T and n but f_5 has a somewhat different T in spite of the same n. f_2 and f_4 show a similar behavior. Their fitting temperatures, however, are different.
- For all π spectra, the best fitting is given by f_6, the simple p_T distribution because of its light mass. For heavier particles, the m_T dependence is more explicit.
- Comparing all fitting functions we explored, it tells us that, for all kinds of identified charged particles, $m_T - m$ scaling exhibits the best fitting results.
- Fig. (3) shows that in the elementary collisions, the temperature T linearly depends on the mass of produced charged particles. It is also linearly correlated with the Tsallis non-extensive parameter $q-1 = 1/n$. This is true for all kinds of fitting formulas.
- To investigate the relation between T and $q-1$ we also plot the various fittings into heavy ion collisions. From Fig. (4) one can see their linear dependence as well.

Acknowledgment

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References

[1] C. Tsallis, J. Stat. Phys. 52, 479 (1988) and its refs. For an updated bibliography on this subject, see http://tsallis.cat.cbpf.br/biblio.htm

[2] Here we follow the calculations from lectures by G. Y. Qin in 2015.

[3] C. Y. Wong, "Introduction to High-Energy Heavy Collisions", p24.

[4] T. S. Biro, G. Purcsel, and K. Urmossy, Eur. Phys. J. A 40, 325-340 (2009).

[5] A. Khuntia, S. Tripathy, R. Sahoo and J. Cleymans Eur. Phys. J. A 53, 103 (2017); B. C. Li, Y. Z. Wang and F. H. Liu, Phys. Lett. B 725, 352 (2013); and so on.

[6] S. Acharya, *et al.*, Eur. Phys. J. C. 78, 263 (2018) and its refs.

[7] ALICE Collaboration, Phys. Lett. B 728, 25 (2014) and Phys. Lett. B 758, 389 (2016).

[8] PHENIX Collaboration, Phys. Rev. Lett. 88, 242301 (2002); ALICE Collaboration, Phys. Lett. B 736, 196 (2014) and Phys. Rev. C 91, 024609 (2015).