

PROBABILITY DENSITY-BASED IMAGE RECONSTRUCTION FOR PROTON COMPUTED TOMOGRAPHY

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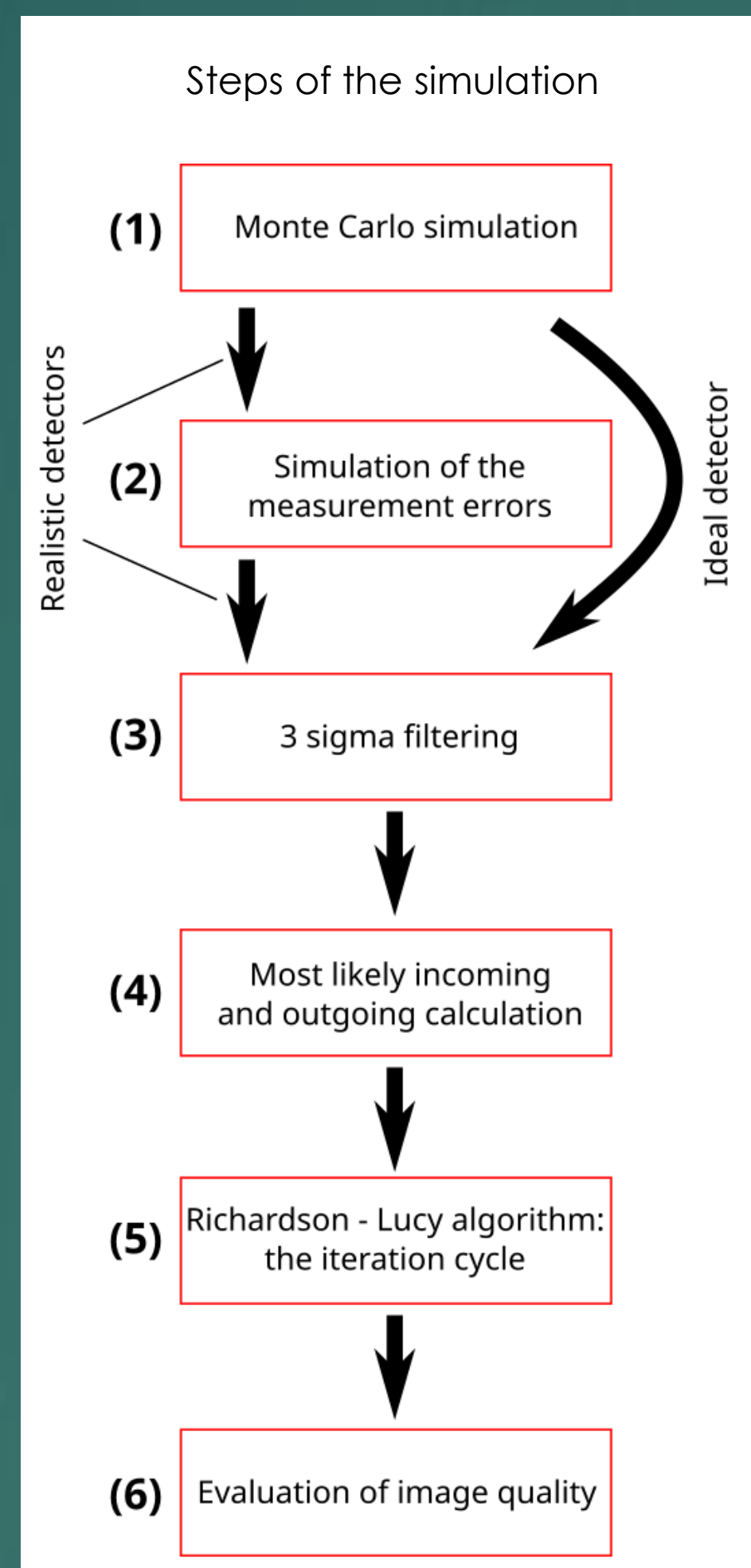
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Objective: Proton therapy is an emerging method against cancer. One of the main development is to increase the accuracy of the Bragg-peak position calculation, which requires more precise relative stopping power (RSP) measurements. An excellent choice is the application of proton computed tomography (pCT) systems which take the images under similar conditions to treatment as they use the same irradiation device and hadron beam for imaging and treatment. A key aim is to develop an accurate image reconstruction algorithm for pCT systems to reach their maximal performance.

Approach: An image reconstruction algorithm was developed in this work, which is suitable to reconstruct pCT images from the energy, position and direction measurement of individual protons. The flexibility of an iterative image reconstruction algorithm was utilised to appropriately model the trajectory of protons. Monte Carlo (MC) simulations of a Derenzo and a CTP404 phantom was used to test the accuracy of the image reconstruction.

Main results: The Richardson – Lucy algorithm was applied first and successfully for pCT image reconstruction. Probability density based approach was applied for interaction (system) matrix generation, which is an advanced approach to consider the uncertain path of the protons in the patient.

Significance: The track of protons are scattered when they travel through material at the hadron therapy energies. This property limits the achievable spatial resolution, especially for single-sided pCT setups investigated in this study. The main motivation of the presented research is to test new approaches for the image reconstruction, focusing on the achieved spatial- and density resolution and the image noise. Realistic imaging setup were simulated with reasonably low proton statistics, to achieve results, which is likely to be reproducible in clinical environment.



METODOLOGY

The pCT literature usually measures the energy of protons in the units of their path length in water, referred as water equivalent path length (WEPL). During the data collection the pCT detector measures the WEPL reduction of the protons. This approach is useful, because the WEPL reduction is independent of the energy of the proton as long as it travels through the same trajectory. The reconstruction volume is divided into volumetric pixels, called voxels. From WEPL reduction point of view the interaction of a proton with consecutive voxels are independent. As a consequence the imaging can be modelled with the following linear equation system:

$$y = A \cdot x,$$

where y vector contains the WEPL reduction of protons, A matrix contains the (expected) path length of every proton in every voxel and x contains the RSP of the voxels. The goal is to restore vector x after the measurement of vector y and matrix A .

The Richardson–Lucy algorithm^{*,**}:

$$x_i^{k+1} = x_i^k \frac{1}{\sum_j A_{ij} x_j^k} \sum_j \frac{y_j}{\sum_i A_{ij} x_i^k} A_{ij},$$

for every $i = 1, \dots, N$, where N is the length of vector x , which contains the RSP of the voxels, k is the number of iterations, matrix A_{ij} contains the interaction coefficients between the proton trajectories and the voxels, $j = 1, \dots, M$ is the index of the trajectories, where M is the number of the trajectories, y contains the integrated RSP along the trajectories, which is equivalent with the WEPL reduction of the protons travelling along the trajectories. The $\frac{y_j}{\sum_i A_{ij} x_i^k}$ term is usually called Hadamard ratio, and it represents the ratio of the integrated RSP along the proton path and its estimate based on the voxel values calculated in the previous iteration.

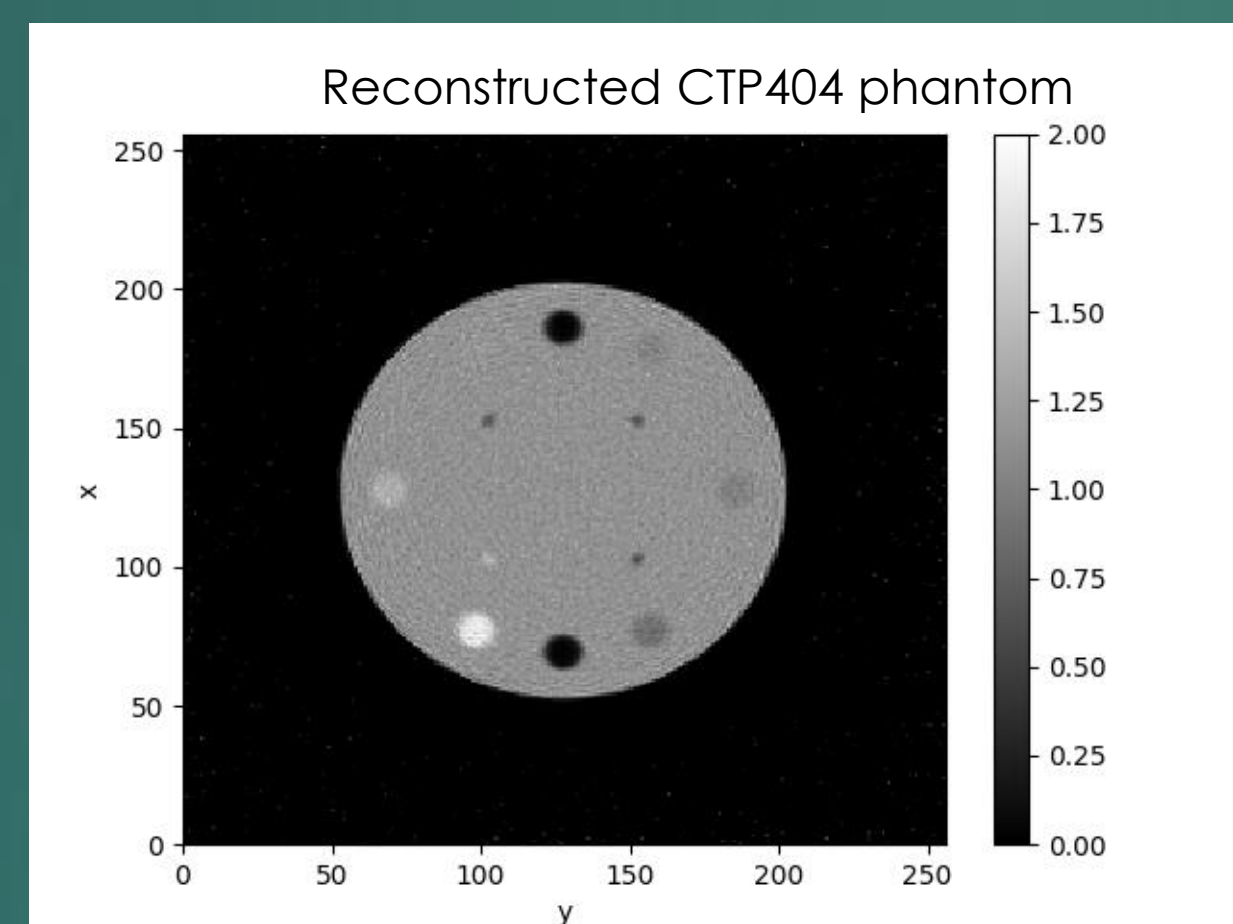
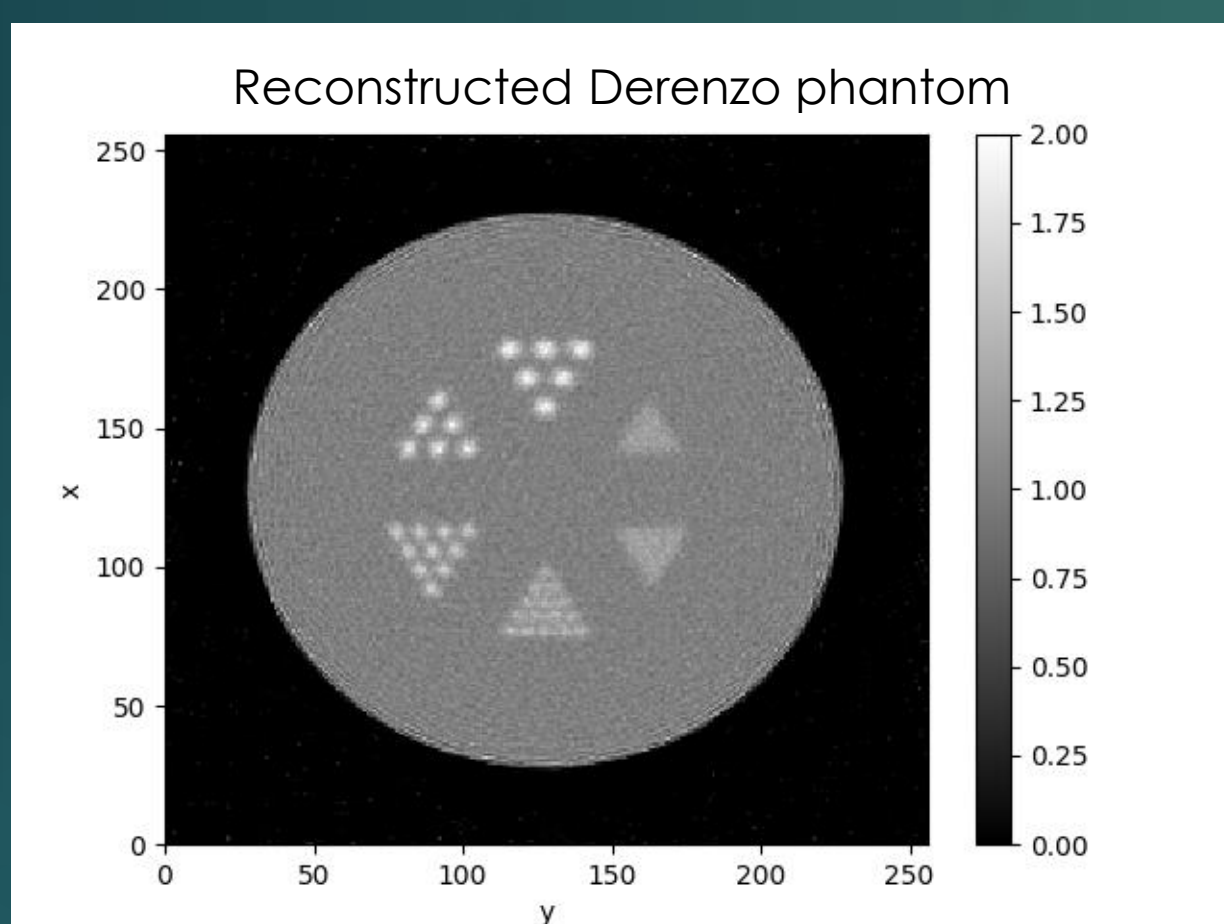
Estimation of the proton trajectory:

The most likely path (MLP) of the protons can be estimated from their incoming and outgoing directions and positions^{***}. In this work the MLP was substituted by a spline approximation to increase computation efficiency. The uncertainty of the trajectory was taken into account by a Gaussian probability density distribution around the MLP of the protons.

*) Richardson, William Hadley (Jan. 1972). "Bayesian-Based Iterative Method of Image Restoration". In: Journal of the Optical Society of America 62 (1). doi: 10.1364/JOSA.62.000055.

***) Lucy, L. B. (June 1974). "An iterative technique for the rectification of observed distributions". In: Astronomical Journal 79. Provided by the SAO/NASA Astrophysics Data System, p. 745. doi: 10.1086/111605.

****) Krahn, Nils, Feriel Khellaf, Jean Michel Letang, et al. (July 2018). "A comprehensive theoretical comparison of proton imaging set-ups in terms of spatial resolution". In: Physics in Medicine and Biology 63 (13). doi: 10.1088/1361-6560/aaca11.



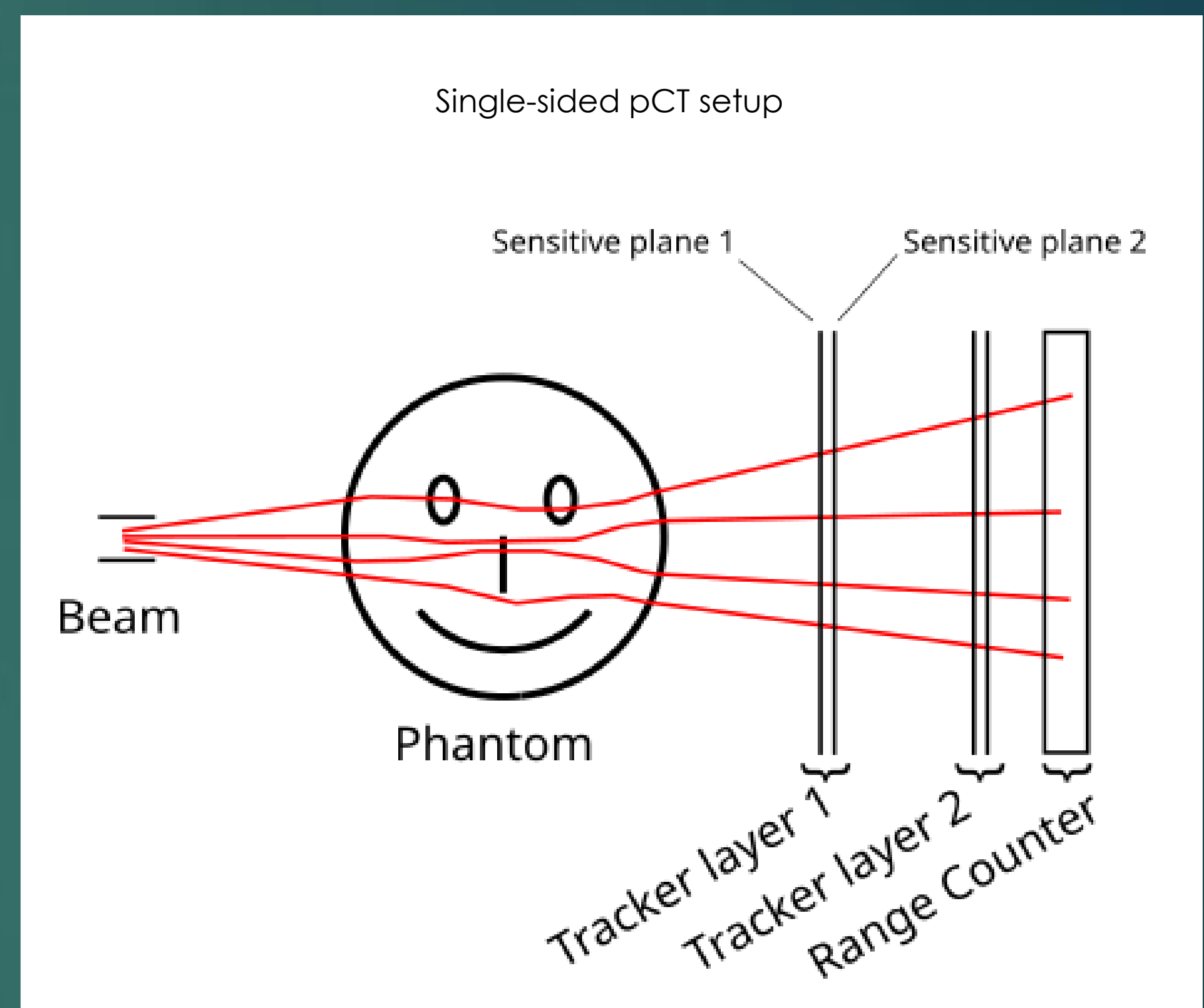
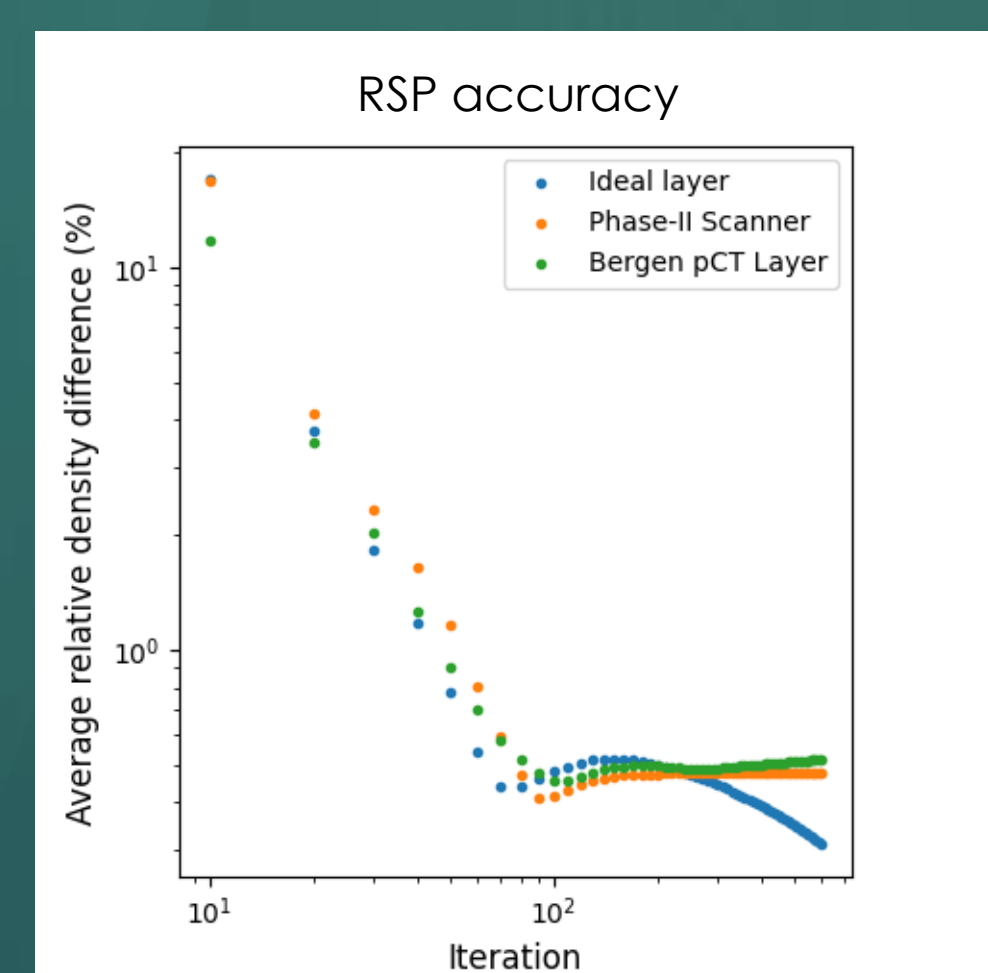
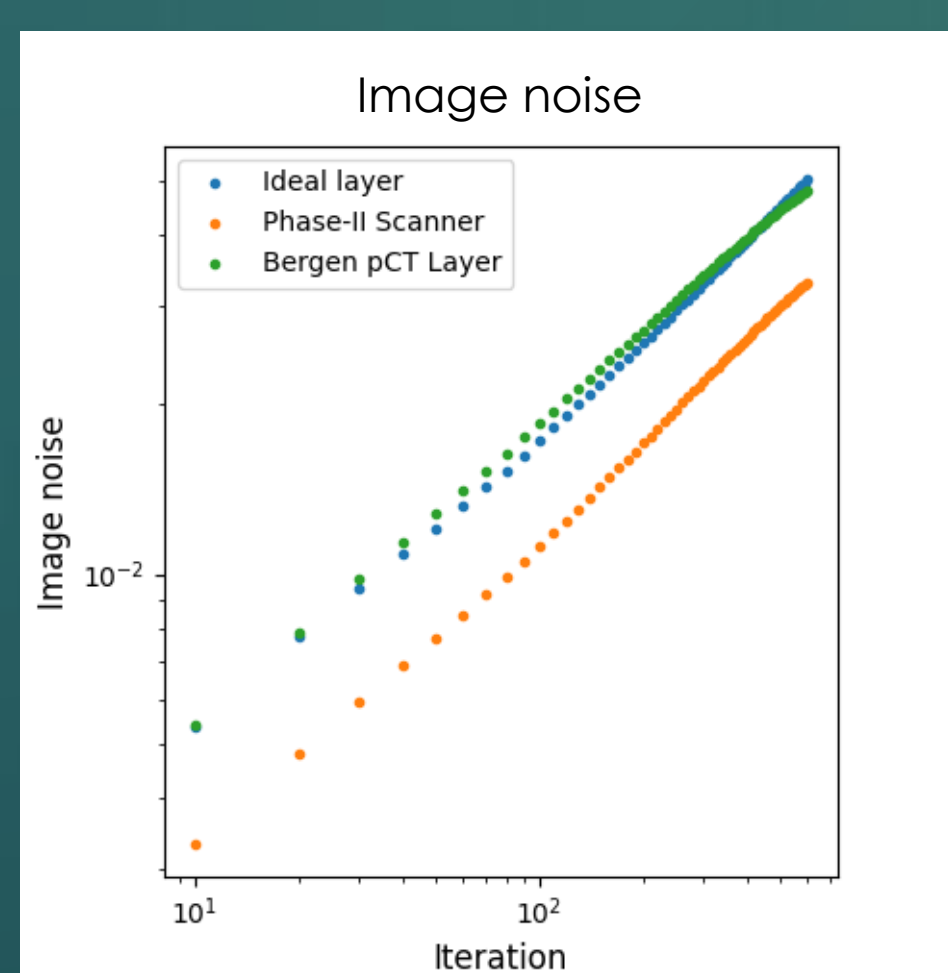
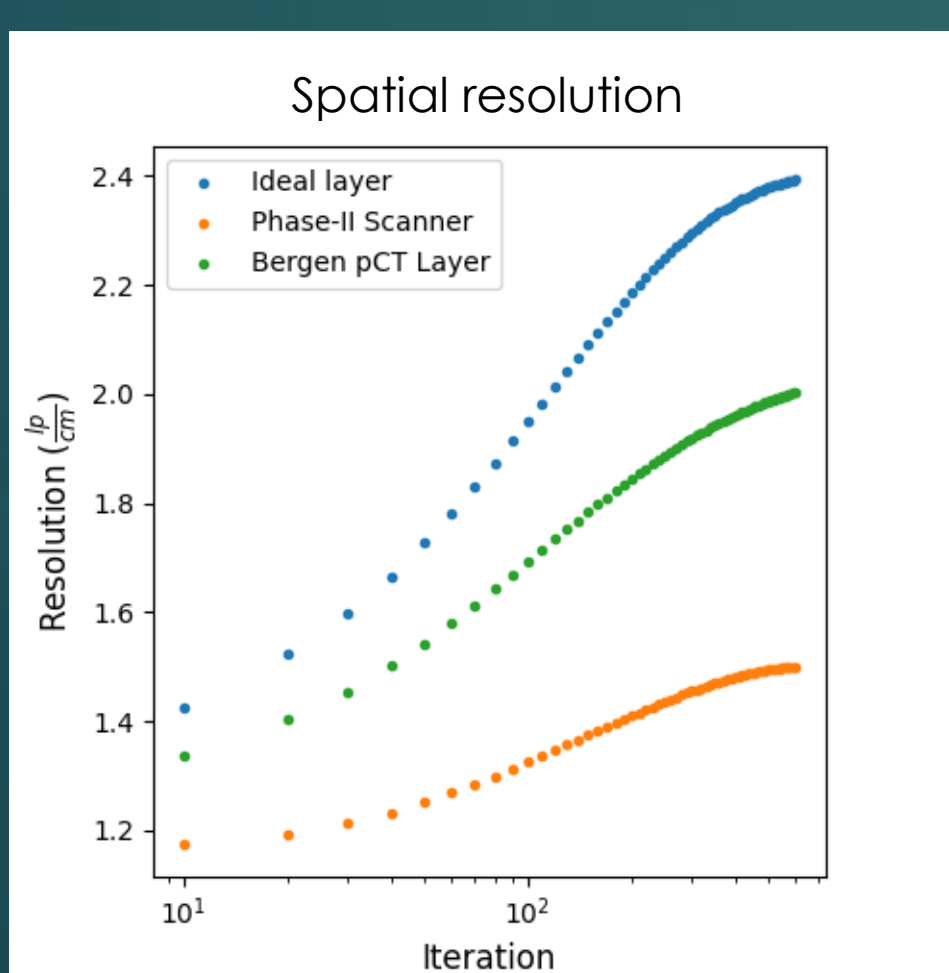
SIMULATIONS WITH THE ALGORITHM

A single-sided detector design with a 230 MeV/u pencil beam was investigated. Three different detector layer was investigated: the first is an idealized detector with no measurement errors, the second is a silicon pixel detector and the third is a silicon strip detector based setup. The data taking was simulated with Monte Carlo method using Geant4 (version 11.0.0) with GATE (version 9.2). In the reference physics list settings QGSP BIC EMY was activated for the calculations. The beam and the phantom were modeled appropriately in the simulation, while the measurement uncertainties were assigned in the next step randomly to the simulated values.

	Unit	Ideal setup	Silicon pixel	Silicon strip
Layer material budget (x/X_0)	-	0	4.2×10^{-3}	8.5×10^{-3}
Distance between layers	mm	-	50	50
Spatial resolution	μm	0	5	66
Angular resolution (130-230 MeV/u)	mrad	0	1.7 – 2.9	3.1 – 4.6
Correlation (130-230 MeV/u)	mradxmm	0	-5×10^{-4}	-8.7×10^{-2}
Statistical WEPL resolution	mm	0	3	3

RESULTS

The spatial and density resolution was tested with dedicated phantoms: the Derenzo and the CTP404 phantoms, respectively. The spatial resolution (left panel) of the ideal setup was found to be 2.4 lp/cm for the ideal setup, 2.0 lp/cm and 1.5 lp/cm for the silicon pixel and the silicon strip detector based setups, respectively. The noise (middle panel) of the ideal and silicon pixel setups seems to be similar (5 % and 4.8 %, respectively), while the silicon strip detector based setup reached significantly lower noise (3.3 %). The error of the density reconstruction is quickly decreasing up to 70-90 iterations, thereafter all saturates, while for the ideal setup it is getting better at large iteration numbers. The average relative RSP difference (except air) was found to be 0.3 % for the ideal and 0.5 % for the realistic setups after 600 iterations. The RSP of the air instead of the real 0.001 was found to be 0.036, 0.051 and 0.061 for the ideal, the silicon pixel and the silicon strip setups. The density resolution was found to be significantly better than the required 1 % RSP accuracy, except of the low density region. The spatial resolution approached, but did not reach the expected 2.5 lp/cm.



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