# Mass Scaling in the Non-extensive Hadronization Model

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## Motivation

Experimental observations suggest that, inclusive **hadron yield follows Tsallis – Pareto-like distribution**, which can be fit well by the formula,

$$\frac{\mathrm{d}^2N}{p_T\mathrm{d}p_T\mathrm{d}y} = A\cdot m_T\cdot\left[1+\frac{E}{nT}\right]^{\frac{1}{1-q}}, \qquad (1)$$

where $A$ is the normalization factor, $m_T=\sqrt{p_T^2+m^2}$ is the transverse mass, $E=\gamma(m_T-vp_T)-m$ is the one-particle energy in the co-moving coordinate system, $\gamma=1/\sqrt{1-v^2}$ is the Lorentz-factor, $T$ is a parameter with temperature unit and finally $q$ is the non-extensivity parameter, characterizing the temperature fluctuations. Fit parameters were found to have **scaling with $\sqrt{s}$ and mass hierarchy with hadron mass,** $m$ [1].

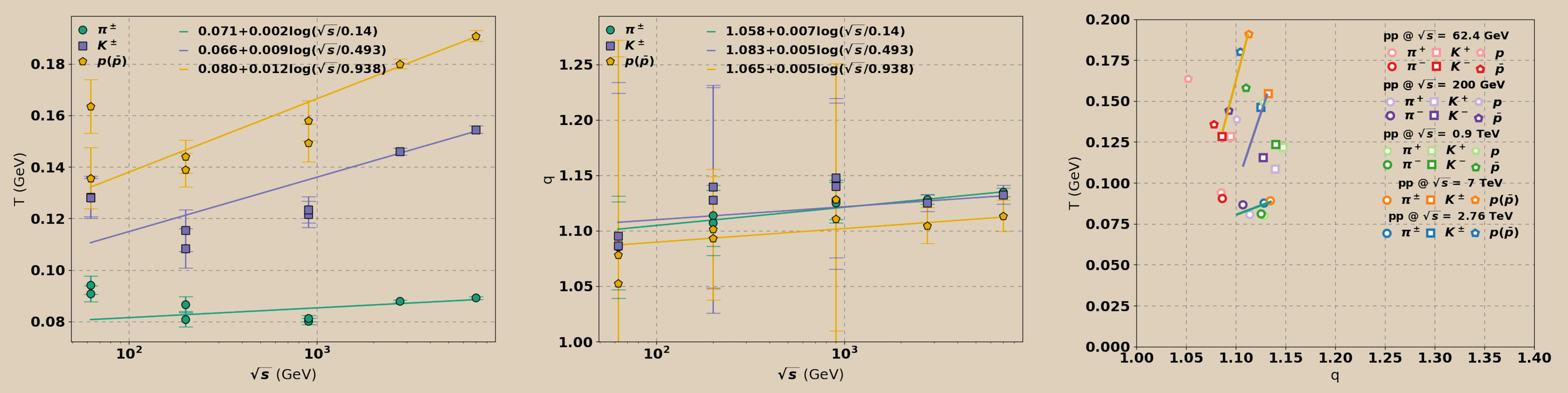

Fig. 1: Parameter scaling based on pp data Tsallis – Pareto fit and the Tsallis-thermometer.

Branches on the **Tsallis-thermometer** ($T-q$ diagram), suggest that description with non-extensive entropy is strongly motivated indeed QCD-like strong correlations, since

(i) parton/hadron **mass play role at low-momentum**, during hadronization,
(ii) parameters evolve by **QCD-like scaling with** $\sim\log Q^2$ **at high-momenta**.

We investigated if we can see similar scalings in the simpest QCD-based hadronization scheme, the **parton fragmentation** model at parton-channel level, following Ref.[2].

## Hadronization by fragmentation functions

In Feynman – Filed fragmentation model the probability of partons confine into hadrons can be given by the **fragmentation functions**, $D_i^h$ describes the probability of a parton, $i$ forms a hadron, $h$. At leading order (LO), $e^-+e^+\to h+X$ spectra is calculated by

$$\frac{1}{\sigma_{tot}}\frac{\mathrm{d}\sigma(e^-e^+\to hX)}{\mathrm{d}z} = \frac{1}{\sigma_{tot}}\sum_i \sigma_0^i(s)D_i^h(z,Q^2), \qquad (2)$$

where the partonic $2\to2$ cross sections are denoted by $\sigma_0$, the energy scale is $Q=\sqrt{s}/2$, and energy fraction is $z=E_h/Q$. The Dokshitzer – Gribov – Lipatov – Altarelli – Parisi equation provides the scale evolution of the fragmentation functions at any $Q$ values,

$$\frac{\mathrm{d}D_i^h(z,\mu)}{\mathrm{d}\log Q^2} = \sum_j\int_z^1\frac{\mathrm{d}x}{x}P_{ij}\left(z/x,Q^2\right)D_i^h\left(x,Q^2\right). \qquad (3)$$

Fragmentation function is a parametrized phenomenological function, providing the probability of a parton $i$ confine into a hadron $h$. The form is motivated by measured spectra.

### The standard QCD-motivated, polynomial approximation [4, 5]

$$D_i^h(z,Q) = N_i^h z^{\alpha_i^h}(1-z)^{\beta_i^h} \qquad (4)$$

- There is **no physical meaning of parameters**: $N,\alpha$, and $\beta$.
- Theoretical predictions for $N,\alpha$, and $\beta$ differs from the experimental data.
- The $Q$-evolved formula, by DGLAP equation is NOT polynomial like.

### New method: non-extensive Tsallis - Pareto-like fragmentation function [6]

$$D_i^h(z,Q) = N_i^h(1-z)\left[1-\frac{q_i^h-1}{T_i^h}\log(1-z)\right]^{\frac{1}{1-q_i^h}} \qquad (5)$$

- **Parameters carry physical meaning** in the non-extensive statistical approach [3]:
  - $q\neq1$ domination of correlations and fluctuations inside the hadronizing system,
  - $T$ temperature of the highly-correlated hadronizing system.
- Theoretical predictions can be given by the non-extensive phenomena. Depending on energy and system-size parameters are: $q=1-2$ and $T=1-10^3$ MeV.
- The $Q$-evolved formula, by DGLAP equation is NOT Tsallis - Pareto like.

## References

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## Results with Tsallis-like fragmentation functions

### Features of the Tsallis – Pareto-like parametrizations

A new parametrization for fragmentation function is fitted by $e^-+e^+\to\pi+X$ data measured at $\sqrt{s}=91.2$ GeV from Ref. [5-8]. The Tsallis – Pareto-like parametrization:

- relies on statistical physics motivated form,
- provides better $\chi^2$ for the fits,
- works better at small-$z$ regions too,
- needs no further parameters for better fit,
- requires no extra constrains during fits,
- has parameters with physical meaning!

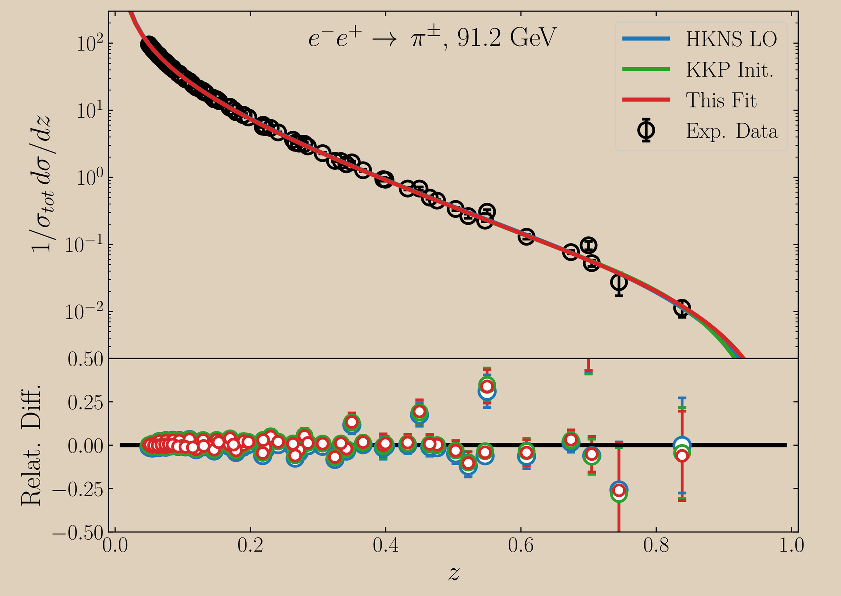

Fig. 2: Pion spectra caclulated as the function of momentum fraction, $z$ using HKNS [4], KKP [5] and the Tsallis–Pareto-like fragmentation functions.

### Parametrized, non-extensive fragmentation functions

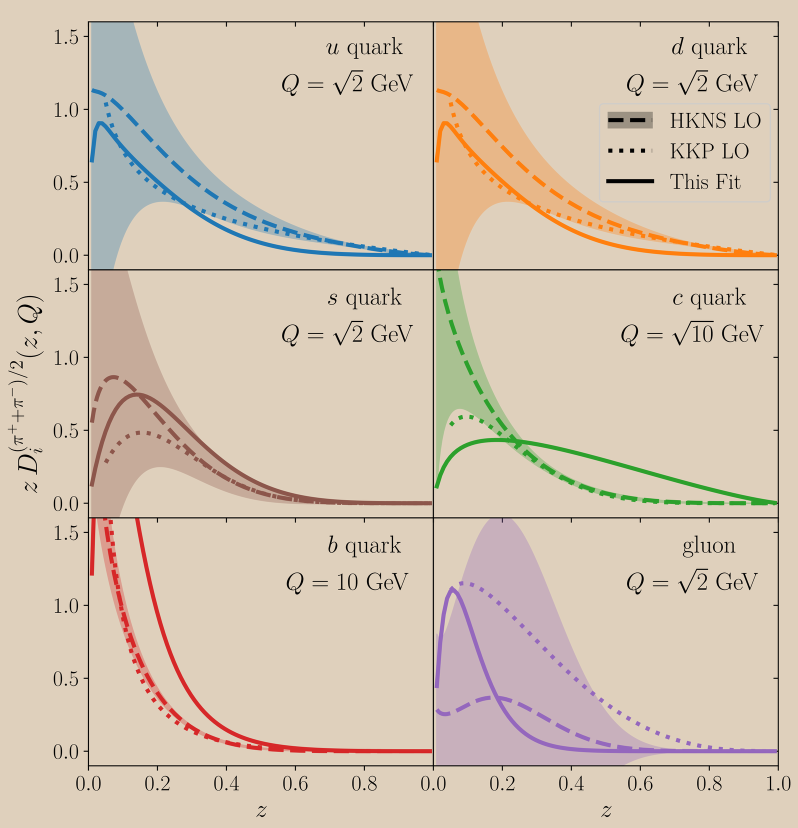

Fig. 3: The Tsallis – Pareto-like fragmentation functions in comparison to HKNS [4] and KKP [5] parametrizations at small $Q$ persents good agreement.

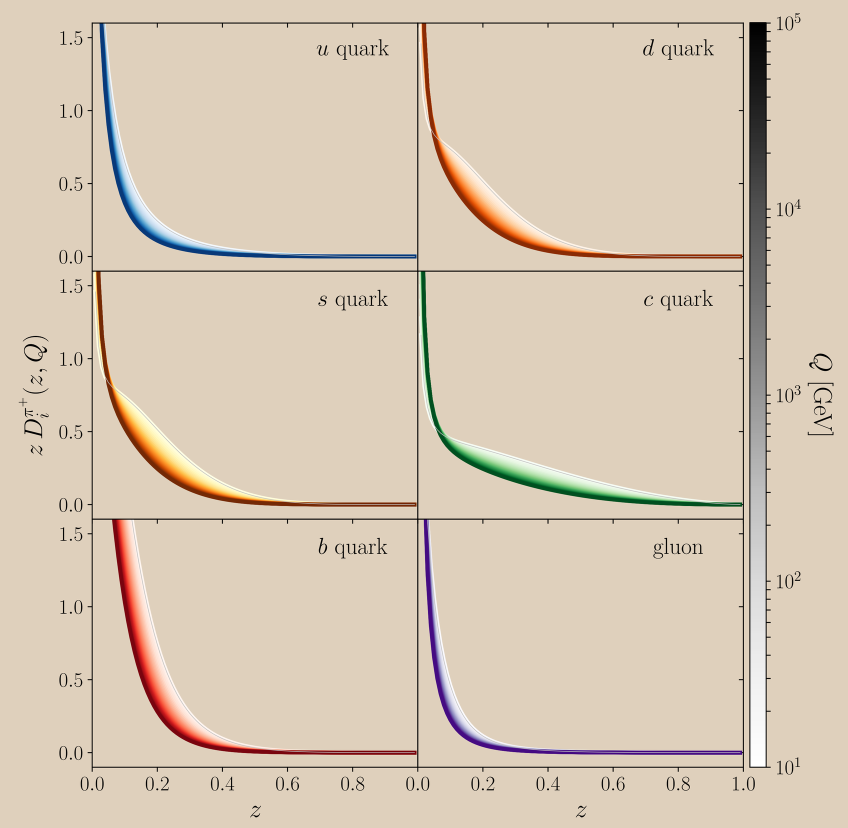

Fig. 4: $Q$ energy scale evolution of our new fragmentation functions by solving DGLAP equations numerically. At high $Q$ the initial differences are vanishing.

### Parameter evolution on the Tsallis-thermometer

The evolution of the $q$ and $T$ parameters on the Tsallis-thermometer present good agreement with the experimental data. Qualitatively, this is similar to the parameter-space evolution of the HKNS [4], KKP [5] models' parameters, $\alpha$ and $\beta$. Agreement between *black curves* and data is good within the proper kinematical ranges, while channel-by-channel contributions can be different.

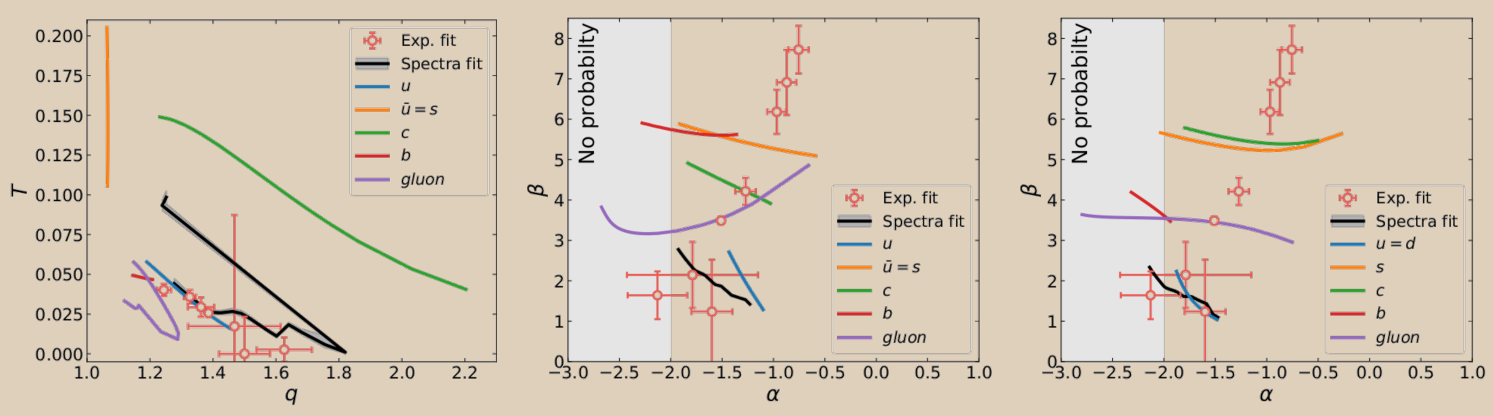

Fig. 5: The Tsallis-thermometer and the paramater space evolution of the fragmentation functions.

We found similar scaling for both the polynomial and Tsallis-like parametrization, on the other hand, parameters on the Tsallis-thermometer are more consistent with the data measured proton-proton collisions. We found weak and model dependent mass hierarchy and strong DGLAP-evolution for any parametrizations agreement with the data fits.

## Conclusion

Non-extensive, Tsallis – Pareto-like fragmentation function parametrizations were developed and used to check mass-hierarchy and QCD-like $\sqrt{s}$ scaling:

- Tsallis-like form and parameters are well physical-motivated,
- Experimental data and DGLAP-evolution of the parameters are consistent.
- Mass scaling seem to appear, but strongly model dependent.

## Contacts & Acknowledgements

**Acknowledgements:** This work is supported by the NKFIH OTKA K120660 & K123815 grants, THOR CA15213 COST action, and the Wigner GPU Laboratory.

**See more in Refs: arXiv:1811.01974 & arXiv:1710.09062**