Multiplicity Scaling in Small Systems within the Non-extensive Statistical Approach

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Introduction

The non-extensive statistical description of the identified final state particles measured in high energy collisions is well-known by it's wide range of applicability. However, there are many open questions that need to be answered, including the question of the observed mass scaling of massive hadrons or the size and multiplicity dependence of the model parameters. This latter is especially relevant, since the amount of the available experimental data with high multiplicity at small systems is very limited so far.

In this contribution the role of the size of the colliding system and multiplicity dependence of the parameters in the non-extensive hadronization model is investigated with HIJING++ calculations. We present cross-check comparisons of HIJING++ with existing experimental data to verify it's validity in our range-of-interest, as well as calculations at high-multiplicity regions where we have insufficient experimental data.

HIJING++ tuning process

HIJING++ is based on the convolution of sequential collisions of nucleon–nucleon pairs in each nucleus–nucleus interaction → important to have a solid proton–proton collisions baseline.

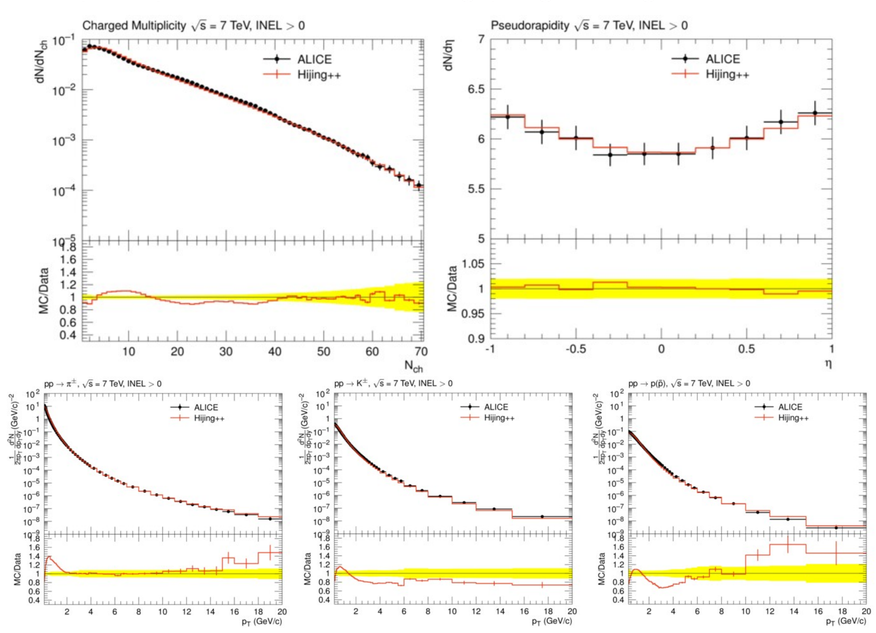
The following √s = 7 TeV proton–proton ALICE experimental data are used:

- p_T spectra of identified charged pions, kaons and (anti)protons with INEL>0 normalization (at least one charged particle in the |η| < 1.0 region is required) up to p_T = 20 GeV/c
- charged hadron multiplicity distribution in the range of ⟨dNch/dη⟩ = 0-70, where N_{ch} is the number of charged particles
- charged hadron pseudorapidity distribution at mid-pseudorapidity |η| < 1.0

HIJING++ has several (~20) phenomenological parameters to be tuned, such as:

Parameter	Description
p_0	soft-hard separation scale: minimum p_T transfer of hard or semihard scatterings
σ_{soft}	the inclusive cross section for soft interactions
σ_0	the cross section that characterizes the geometrical size of a nucleon
μ_0	the parameter in the scaled eikonal function of nucleon used to calculate total cross-section
K	K-factor for the differential jet cross sections in the lowest order pQCD calculation

- HIJING++ reproduces the event multiplicity well within ~15% for the multiplicity and ~1% for the pseudorapidity distribution
- The charged pion and kaon spectra also show a good agreement above p_T = 2 GeV/c
- The production is slightly overestimated at lower p_T values
- Best agreement for the π ± results is ~1% between 2 and 15 GeV/c
- For kaons, the yield is slightly underestimated above 2 GeV/c: the agreement is ~15–20%
- The proton yield is overestimated in the large pT region, the agreement is ~20–30%



Non-Extensive Hadronization

Our goal is to study the transverse momentum spectra of identified hadrons with Tsallis – Pareto type distributions, revealing non-trivial dependence on the center-of-mass energy, the hadron mass and the event multiplicity (i.e. on the size of the system).

- Blast-wave assumptions: the fireball is azimuthally symmetric and is expanding with a v radial flow velocity (in units of c = 1)
- The freeze-out occurs instantly on a hypersurface according to the Cooper – Frye formulation at a given freeze-out temperature
- The following simple form of the invariant yield is used:

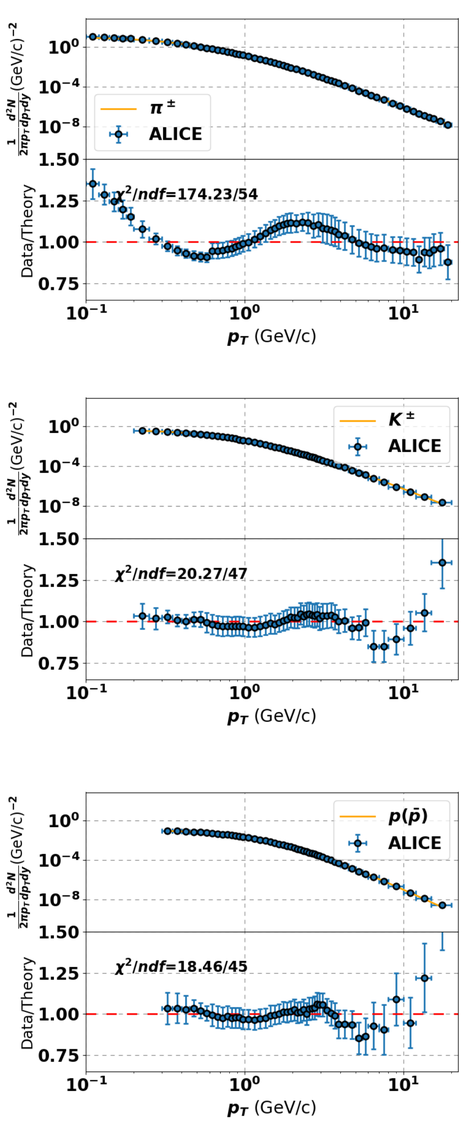
$$\left.\frac{d^2N}{p_T dp_T dy}\right|_{INEL>0} = A \cdot m_T \cdot \left(1 + \frac{E}{nT}\right)^{-n}$$

where A is the amplitude incorporating the irrelevant spin degeneracy and constant factors as well as the invariant volume, $m_T = \sqrt{p_T^2 + m^2}$ is the transverse mass, $E = \gamma(m_T - vp_T) - m$ is the one-particle energy in the co-moving coordinate system, *v* is the radial flow velocity, $\gamma = 1/\sqrt{1-v^2}$ is the Lorentz-factor, *T* is a parameter with temperature unit and finally *n=1-q* is the non-extensivity parameter, characterizing the temperature fluctuations.

- The parameters and curves fitted on the experimental "minimum bias" (in the sense that there is no event multiplicity classification) data
- The fit is good on the investigated p_T range for all hadrons

Hadron	*n*	*q*	*T* (GeV)	*A*	*v*	χ^2/ndf
$\pi^{\pm}$	7.415 ± 0.033	1.135 ± 0.005	0.089 ± 0.010	73.188 ± 9.700	0.000 ± 0.119	174.225 / 54
$K^{\pm}$	7.539 ± 0.086	1.133 ± 0.013	0.155 ± 0.010	0.915 ± 0.095	0.000 ± 0.066	20.274 / 47
$p(\bar{p})$	8.805 ± 0.184	1.114 ± 0.023	0.191 ± 0.012	0.124 ± 0.013	0.000 ± 0.054	18.462 / 45

The results are consistent with our earlier observations: amont the investigated hadron species, the proton (which is the heaviest) has the largest temperature and the smallest *q* and amplitude. The fitted radial flow velocity is zero for each hadrons.



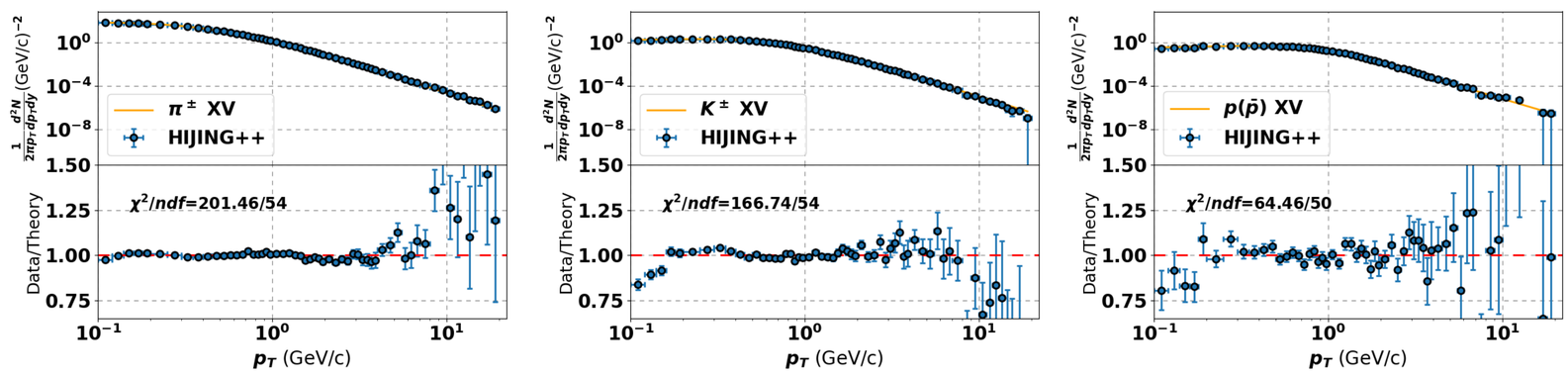
Results

Using the non-extensive hadronization framework we aim to investigate the multiplicity dependence of the Tsallis–Pareto parameters. We take advantage of the power of HIJING++ and extract the parameters from wide range of event multiplicity classes at the tuned √s = 7 TeV center-of-mass energy.

- The event classes of the HIJING++ run are classified as the multiplicity ranges:

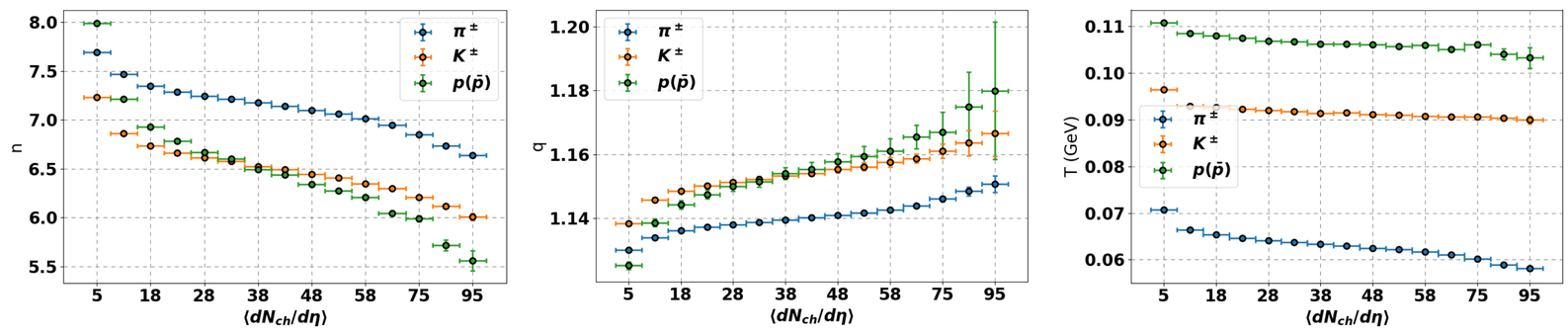
Class	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV
$\langle dN_{ch}/d\eta\rangle_{min}$	0	10	15	20	25	30	35	40	45	50	55	60	70	80	90
$\langle dN_{ch}/d\eta\rangle_{max}$	10	15	20	25	30	35	40	45	50	55	60	70	80	90	100

- Using this event classification, we calculated the mid-rapidity transverse momentum spectra of charge averaged pions, kaons, and protons in INEL > 0 events in the 0.1 GeV/c < p_T < 20 GeV/c region, generating 200M events
- The spectra at the highest multiplicity class along with the fitted Tsallis–Pareto curves are presented:

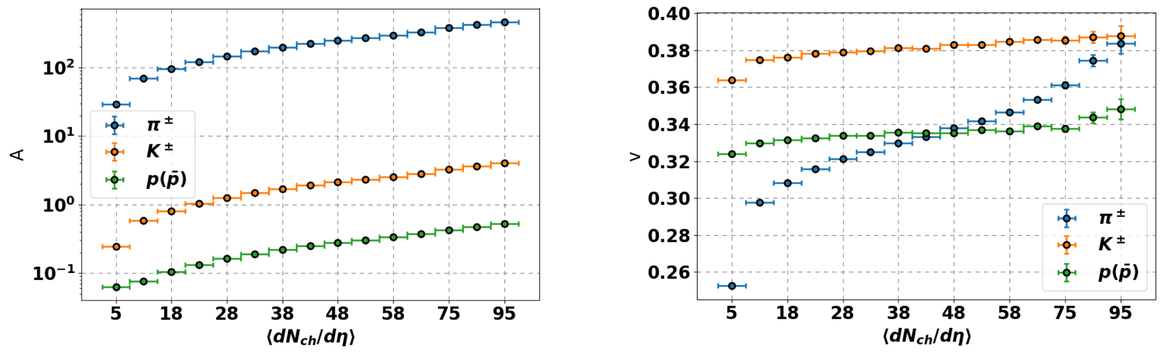


Studying the parameters in the function of the event multiplicity we can draw the following conclusions:

- With increasing multiplicity the *q* parameter increases for each hadron but with different slopes: the increase of *q* (or the decrease of *n*) is the largest for the heaviest hadron
- The temperature decreases slowly with the increasing pseudorapidity density
- The previously observed $T_{\pi^{\pm}} < T_{K^{\pm}} < T_p$ mass hierarchy appears



- The radial flow velocity is not zero and also increases with the multiplicity, but also with different rates: at low multiplicity the pions (with the smallest mass) have the smallest *v*, but it increases rapidly with the increasing multiplicity
- The rate of increase in the case of protons and kaons are approximately the same
- The amplitudes are increasing for each hadron species with the multiplicity
- The value of pions is much higher than those of the heavier hadrons, which indicates that with increasing multiplicity the number of the produced pions grows faster than the number of kaons and protons



Summary

Current status of the tuning process of HIJING++ is presented with √s = 7 TeV ALICE data:

- Good agreement with the main high-energy physics observables such as multiplicity and p_T distributions

Non-extensive hadronization model is used to describe the transverse momentum distribution of identified hadrons:

- Extracted the parameters from √s = 7 TeV proton–proton collisions using ALICE data and tuned HIJING++ with various event multiplicity classifications
- Mass hierarchy emerges in the Tsallis parameters from both experimental and Monte Carlo data
- With increasing multiplicity: increasing *q* non-extensivity parameter and slight decrease of *T* temperature
- In case of HIJING++, all hadrons result in a non-zero, mass depentent, increasing radial flow velocity
- The parameters are sensitive to the event size and may serve as a thermometer of the collision

Acknowledgement

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