Estimating nuclear matter parameters from compact star observables: a traditional method vs. the burte force

G.G. Barnaföldi, in collaboration with
D. Alvarez-Castillo, A. Ayrian, H. Grigorian, A. Jakovác, P. Pósfay, B. Szigeti

Support: *Hungarian OTKA grants, NK123815, K135515 Wigner GPU Laboratory, the PHAROS MP16214 and THOR CA15213 COST actions.*

ELTE TTK ElmFiz Seminar, Budapest, 16th March 2021

Outline

1) Traditional way: mean field model

– In CORE approximation:

using maximal mass compact stars

→ we determine the nuclear parameters & uncertainties

2) Brute force: Bayesian model

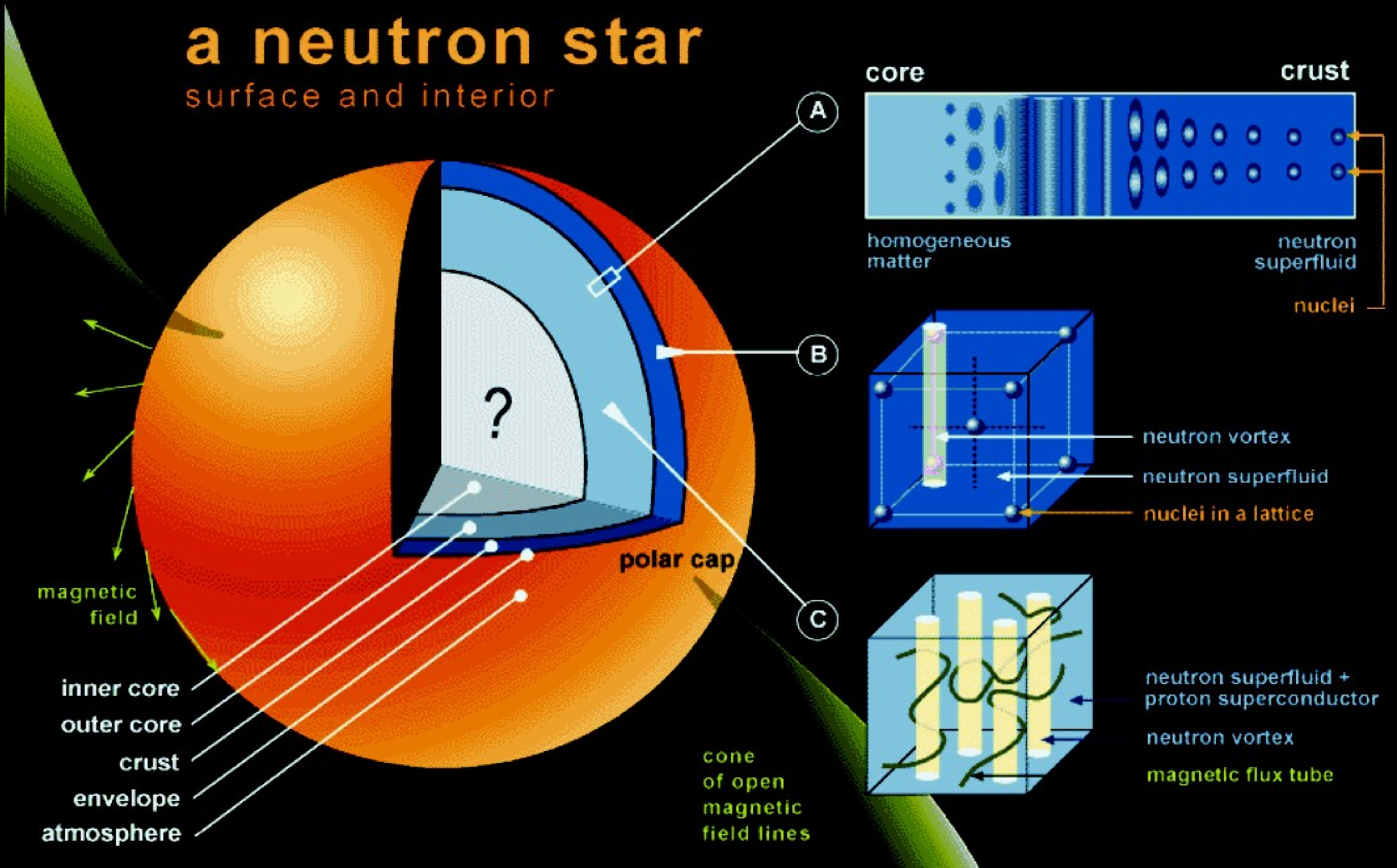
– Maximal mass compact stars + more constraints

from GW data, and M-R observations

→ more precise determination

3) Conclusions:

– Comparison of the traditional & Bayesian results

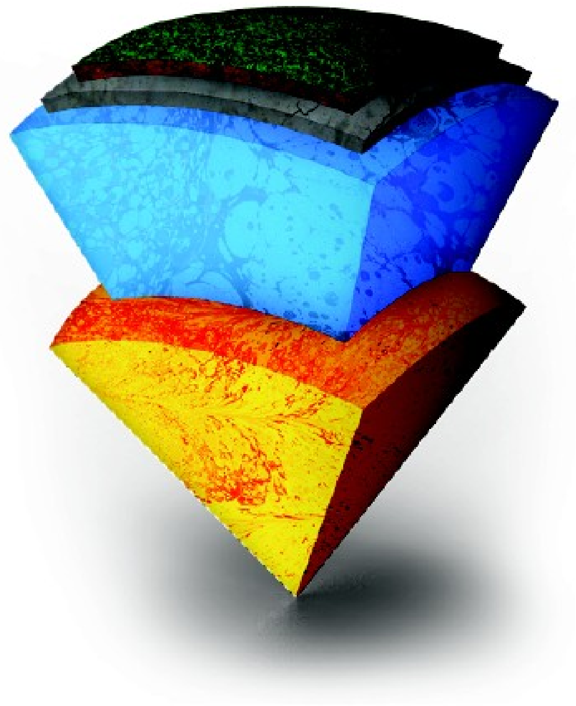

a neutron star
surface and interior
core
crust
A
homogeneous matter
neutron superfluid
nuclei
B
neutron vortex
neutron superfluid
nuclei in a lattice
?
polar cap
C
neutron superfluid + proton superconductor
neutron vortex
magnetic flux tube
magnetic field
inner core
outer core
crust
envelope
atmosphere
cone of open magnetic field lines

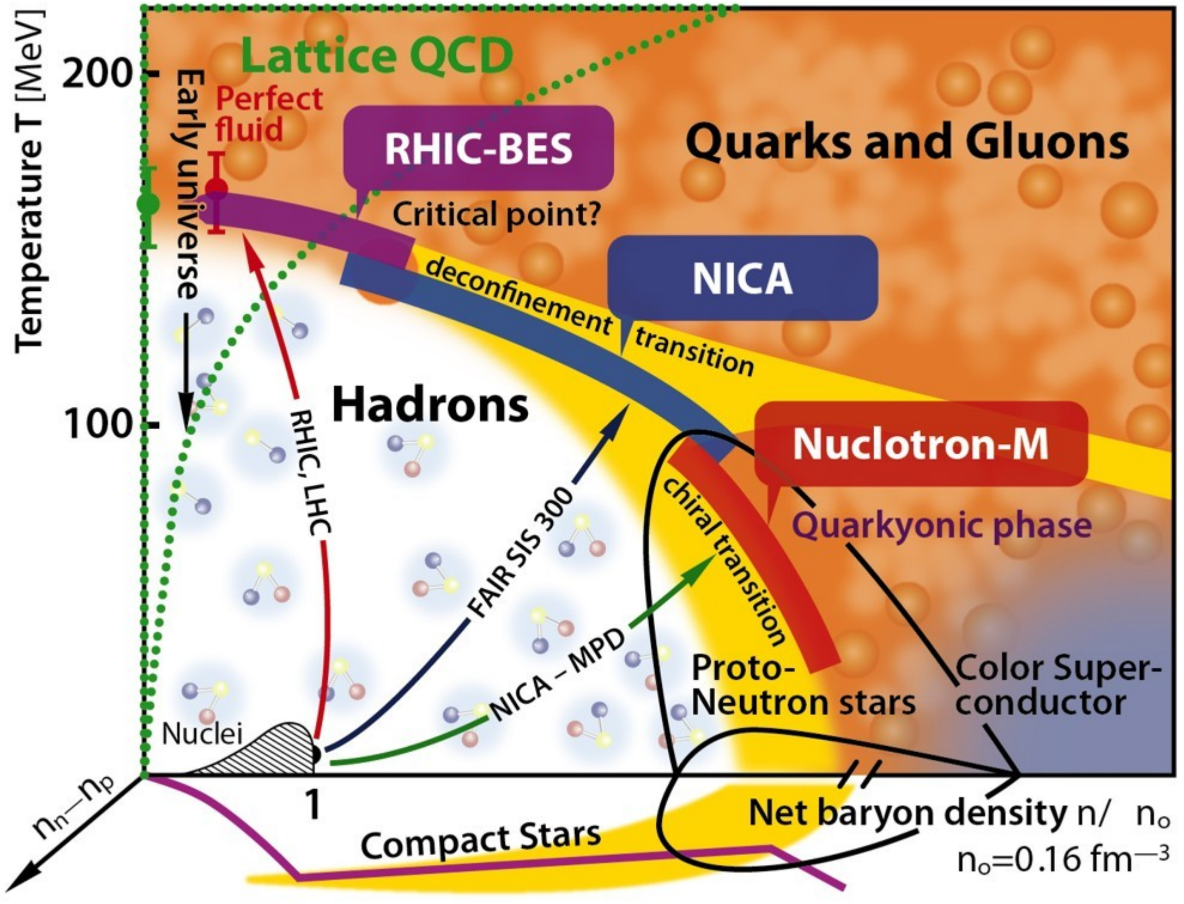
Description of the compact star interior

Dimensions:
R ~ 10 – 15 km
M ~ 1.2 – 2.2 $M_\odot$

Outer crust:
Ions, electron gas, neutrons

Inner core:
Neutrons?
Protons?
Hyperons?
Kaon condensate?
Quark matter?





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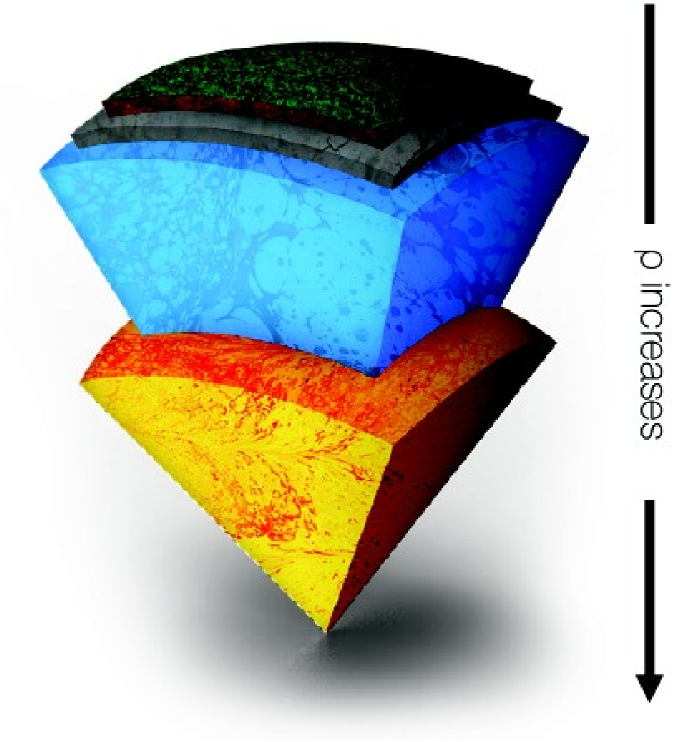
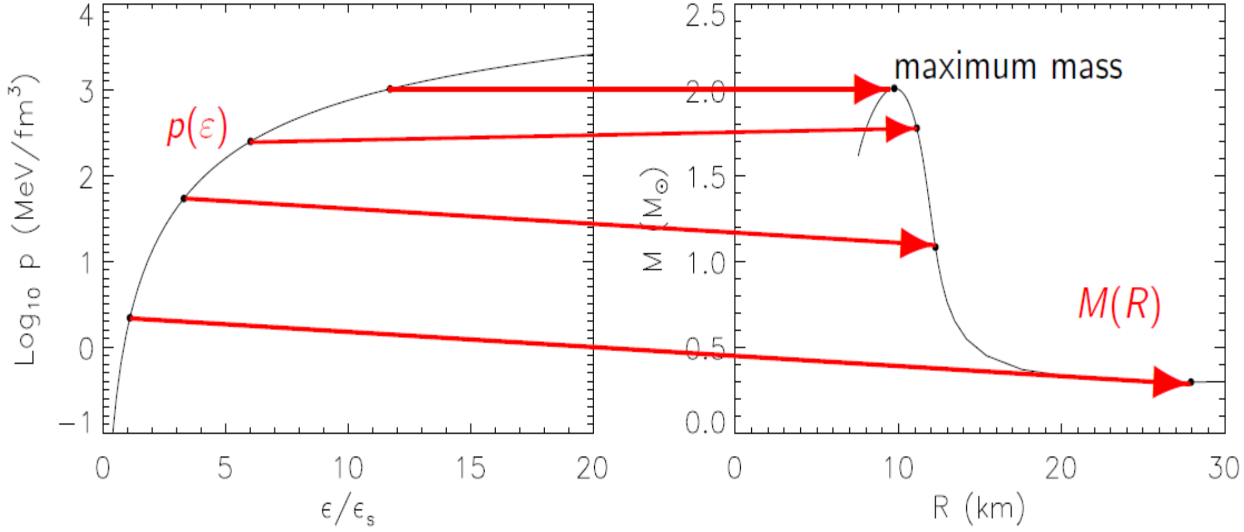


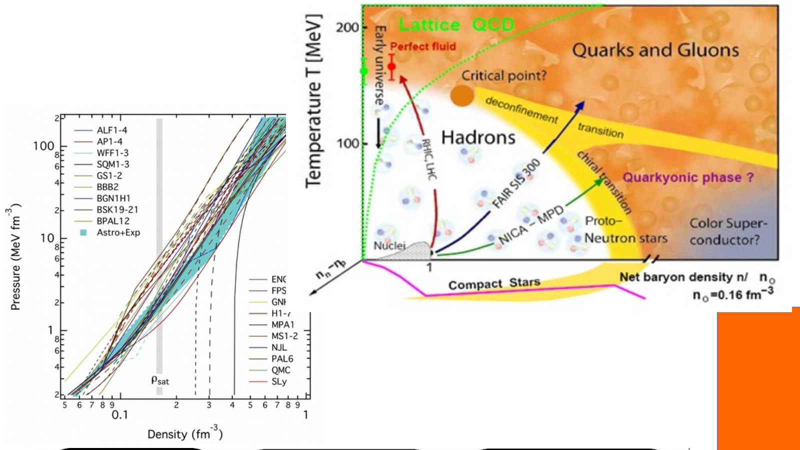
$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi p r^3)(\varepsilon+p)}{r(r-2Gm/c^2)}$$

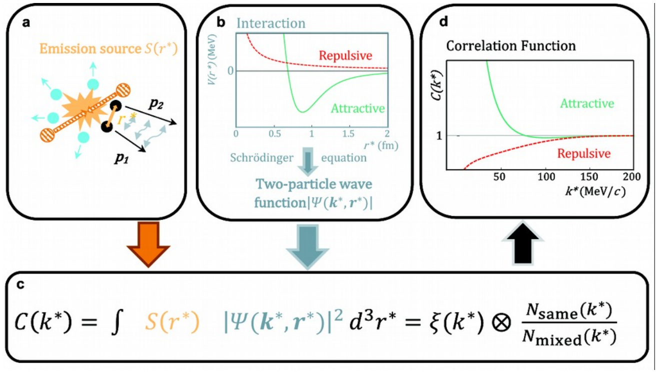
$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



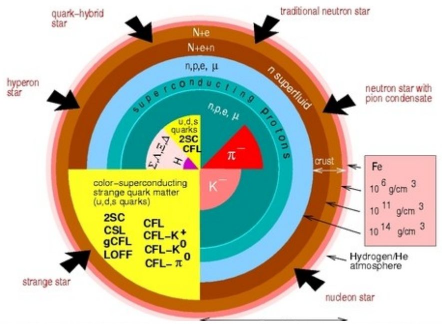
Recent inputs & the program

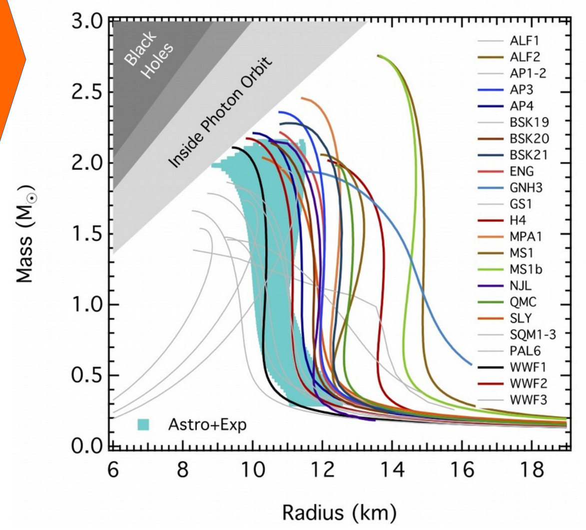
EoS experiment & theory



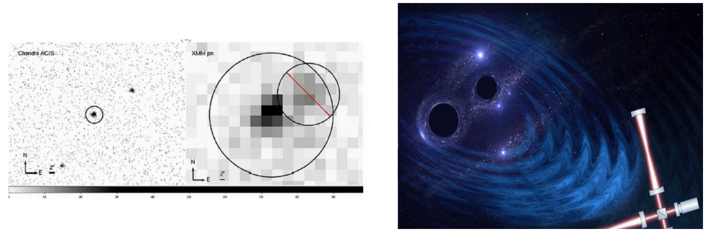


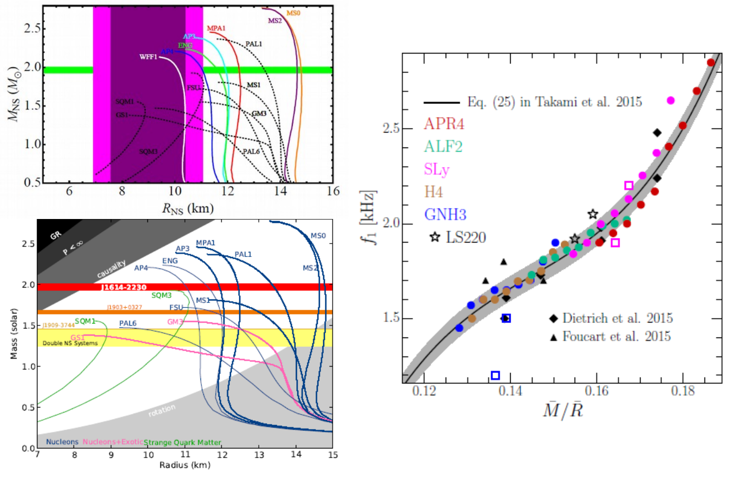
Application in compact stars





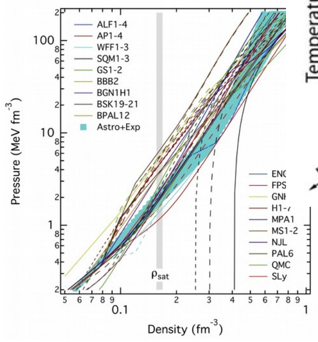
Constraints by astrophysical observations

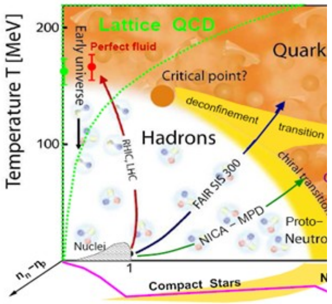


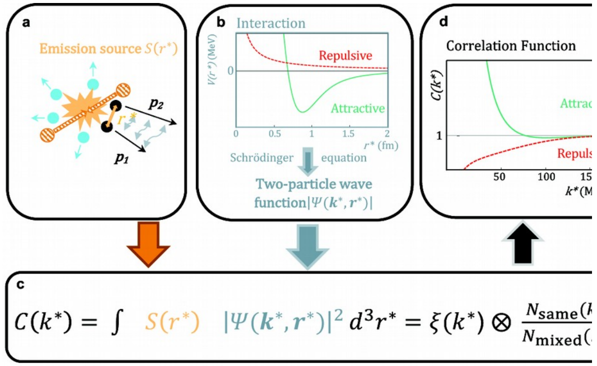


Face the masquarade problem!

EoS
experiment & the







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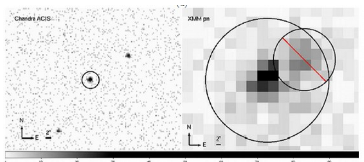
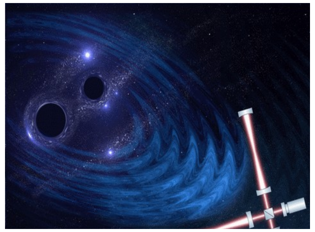
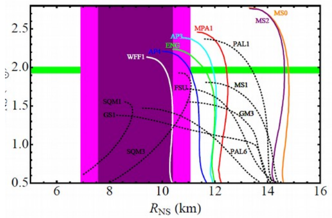
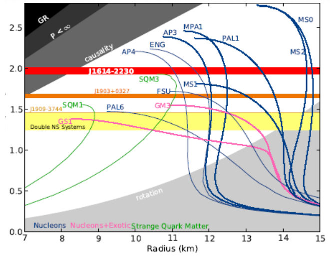
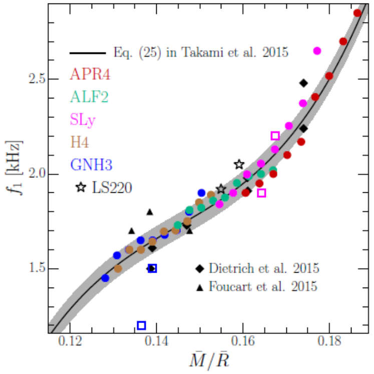




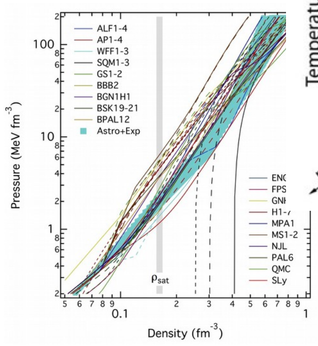


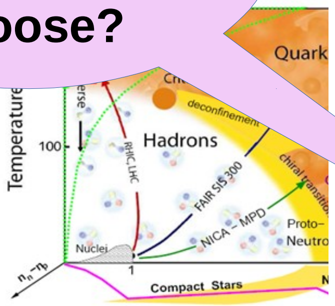


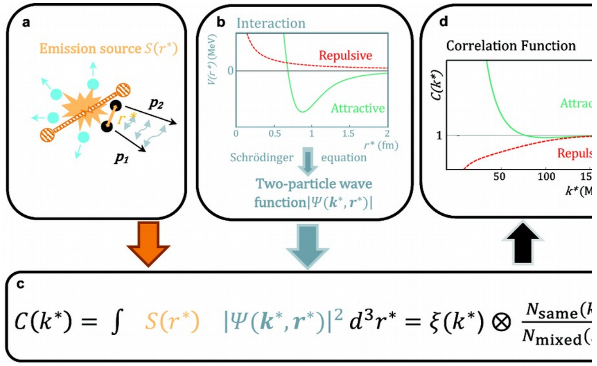


Face the masquarade problem!

Uuups, which EoS I should choose?







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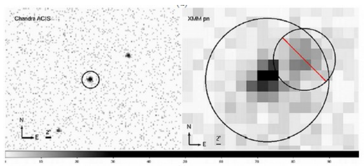
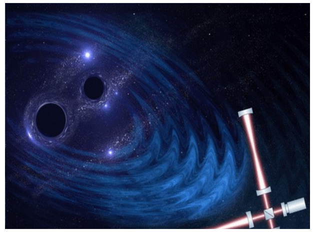
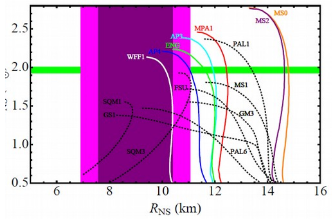
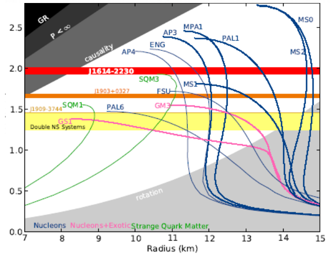
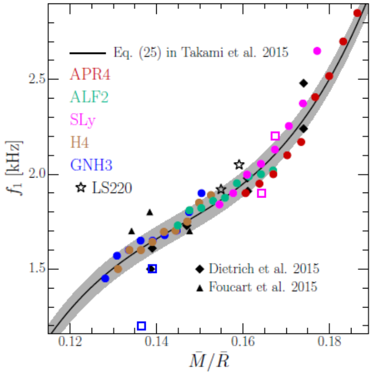




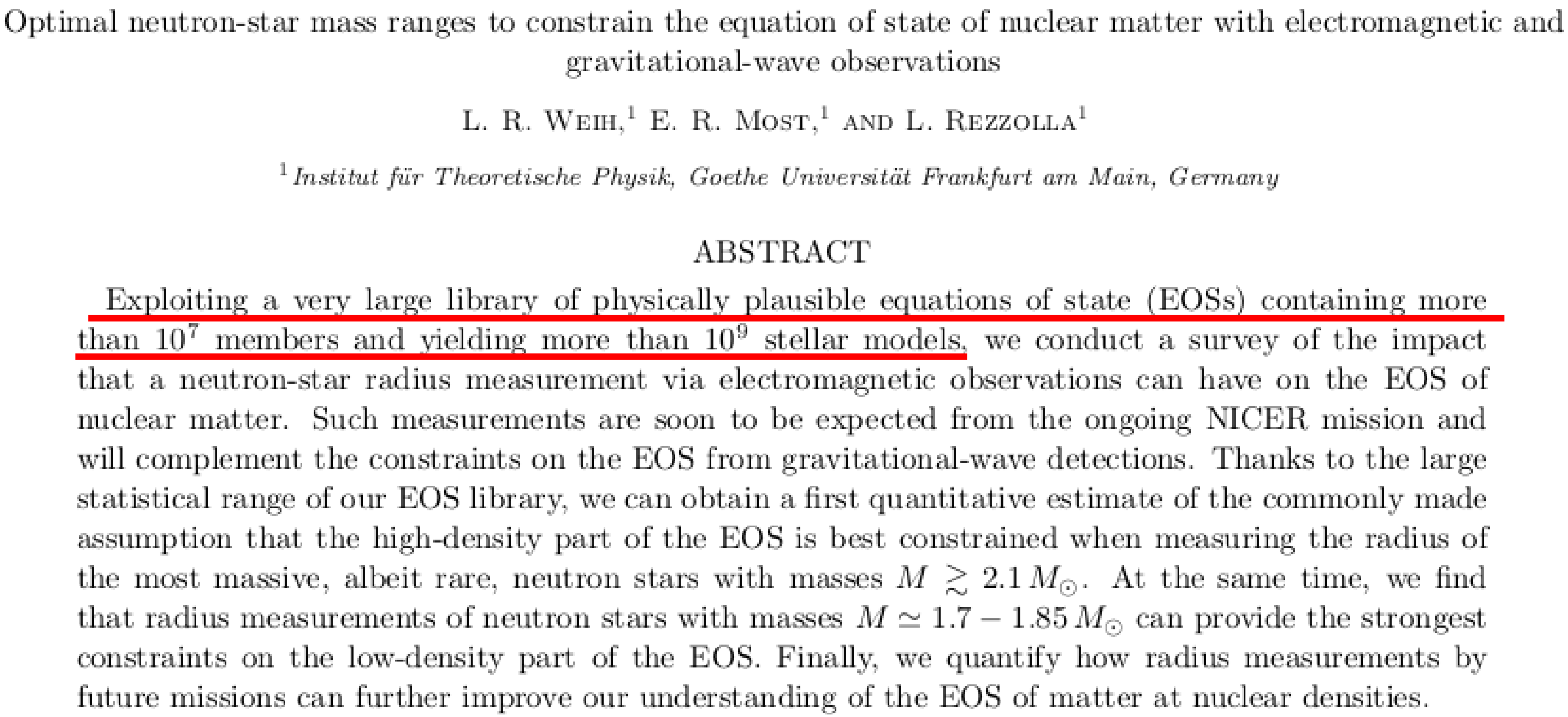






(De)motivation...

Weih & Most & Rezzolla: ApJ 881,73 (2019)


Optimal neutron-star mass ranges to constrain the equation of state of nuclear matter with electromagnetic and gravitational-wave observations

L. R. Weih,[1] E. R. Most,[1] and L. Rezzolla[1]

[1] *Institut für Theoretische Physik, Goethe Universität Frankfurt am Main, Germany*

ABSTRACT

Exploiting a very large library of physically plausible equations of state (EOSs) containing more than 10^7 members and yielding more than 10^9 stellar models, we conduct a survey of the impact that a neutron-star radius measurement via electromagnetic observations can have on the EOS of nuclear matter. Such measurements are soon to be expected from the ongoing NICER mission and will complement the constraints on the EOS from gravitational-wave detections. Thanks to the large statistical range of our EOS library, we can obtain a first quantitative estimate of the commonly made assumption that the high-density part of the EOS is best constrained when measuring the radius of the most massive, albeit rare, neutron stars with masses $M \gtrsim 2.1\,M_\odot$. At the same time, we find that radius measurements of neutron stars with masses $M \simeq 1.7 - 1.85\,M_\odot$ can provide the strongest constraints on the low-density part of the EOS. Finally, we quantify how radius measurements by future missions can further improve our understanding of the EOS of matter at nuclear densities.

The sad reality is...

Weih & Most & Rezzolla: ApJ 881,73 (2

Optimal neutron-star mass ranges to … tter with electromagnetic and

… many

Exploit … containing more
than 10^7 … of the impact
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nuclear mat… mission and
will complem… to the large
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Let’s explore the uncertainties…

…in a traditional way

P. Pósfay, GGB, A. Jakovác: 2004.08230 (submitted to PASA), + B. Szigeti EPJ ST 229 (2020) 3605

Investigate this with extended σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ \underbrace{- m_N + g_\sigma \overline{\sigma}}_{\text{Nucleon effective mass}} - g_\omega \gamma^0 \,\overline{\omega}_0 \right) \psi_i$$

Proton and neutron

$$-\frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \,\overline{\sigma}^3 - \lambda_4 \,\overline{\sigma}^4$$

Scalar meson self interaction terms

Extra terms

$$+\frac{1}{2} m_\omega^2 \,\overline{\omega}_0^2$$

Vector meson

$$+\ \frac{1}{2} m_\rho^2 \,\rho_\mu^a \,\rho^{\mu\, a}$$

Tensor meson

$$+\ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

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Proton and neutron

Nucleon effective mass

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Vector meson

Extra terms

p – n Nuclear force

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$$+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Isospin asymmetry

Tensor meson

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Nucleon effective mass: $m_N - g_\sigma \overline{\sigma}$

Proton and neutron

$$- \frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$$

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$$+ \frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

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Tensor meson

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Vector meson

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Tensor meson

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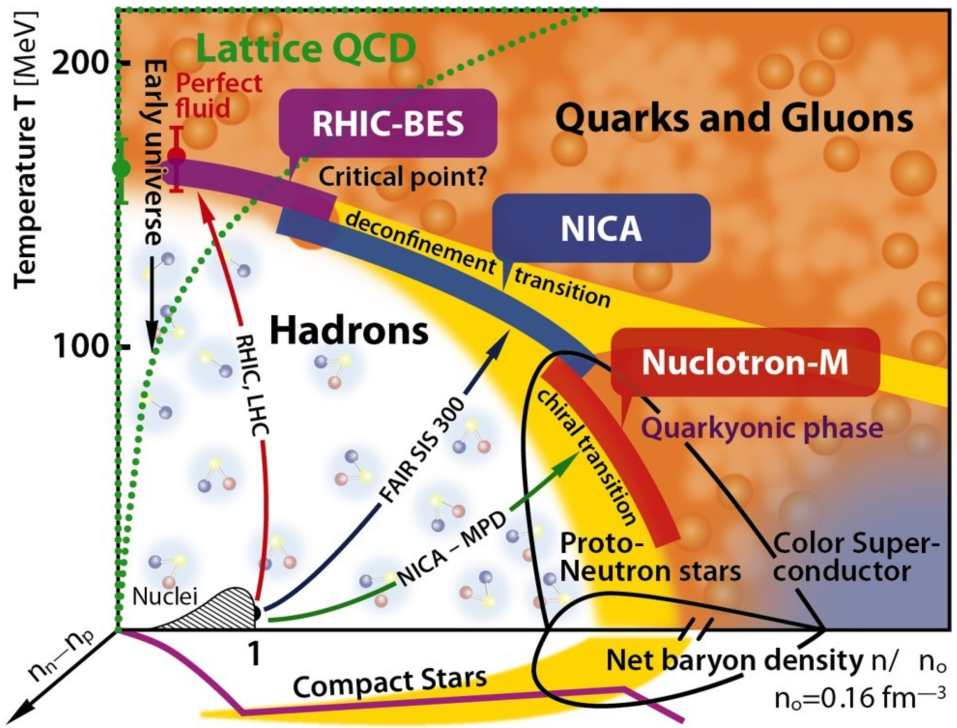
Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

- **Theoretical mean field model:**
  - Symmetric case: 3 combinations with the higher-order scalar meson self-interaction terms to original Walecka:
  - Asymmetric case: tensor force is added to the interaction in addition to the electrons, for β-equilibrium.
- **Parameters of the theoretical model**
  - Fit couplings/masses/etc. according to the Rhoades–Ruffini theorem in agreement with experimental data.
  - Parameters are usually non-independent: optimalization of the parameters need to perform → similar EoS
- **Cross check the consistency with the the existing EM, GR, HIC, etc data + errors → Theoretical uncertainties**

Parameters to fit normal nuclear matter



Model	ns [fm^{-3}]	B [MeV]	K [MeV]	S_0 [MeV]	m^* [m_N]
NLρ	0.1459	−16.062	203.3	30.8	0.603
NL$\rho\delta$	0.1459	−16.062	203.3	31.0	0.603
DBHF	0.1810	−16.150	230.0	34.4	0.678
DD	0.1487	−16.021	240.0	32.0	0.565
D3C	0.1510	−15.981	232.5	31.9	0.541
KVR	0.1600	−15.800	250.0	28.8	0.805
KVOR	0.1600	−16.000	275.0	32.9	0.800
DD-F	0.1469	−16.024	223.1	31.6	0.556

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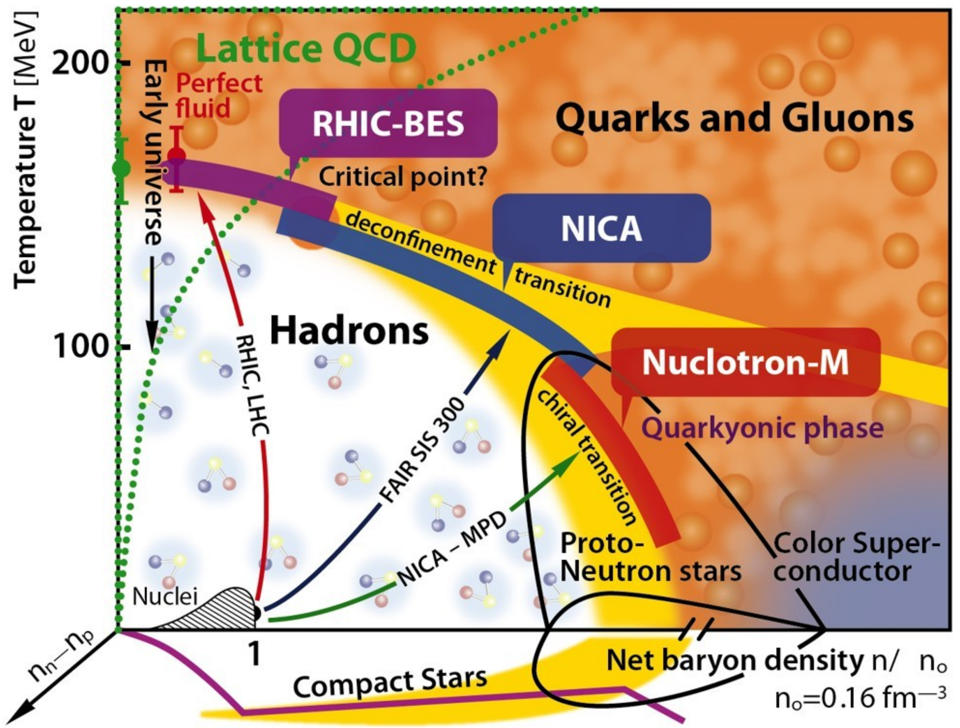
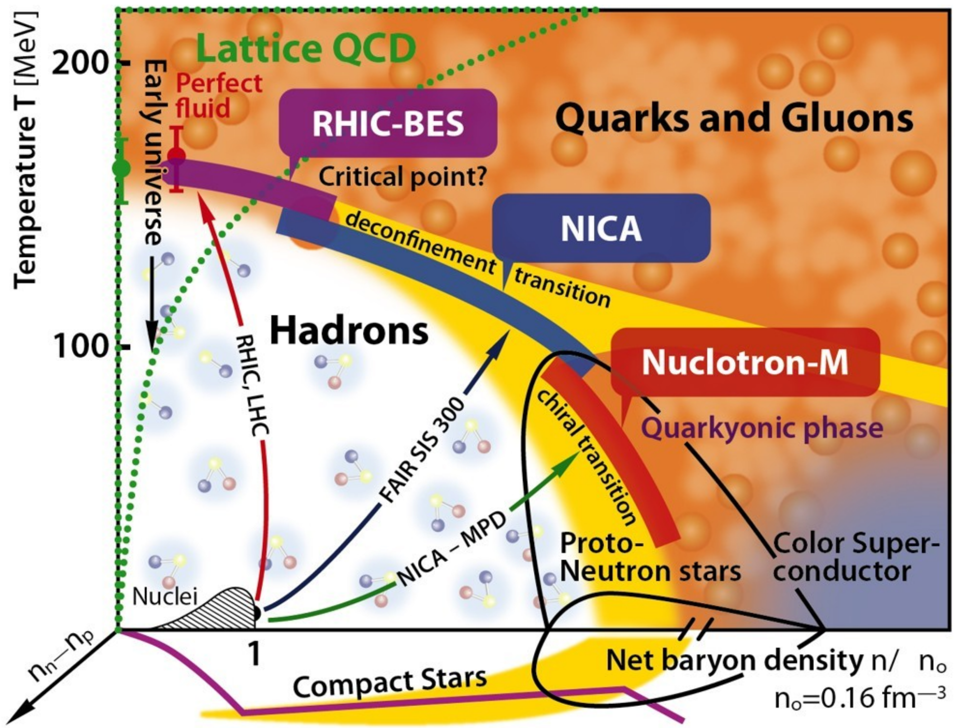


Parameter	Value
Saturation density	0.156 1/fm^3
Binding energy	-16.3 MeV
Nucleon effective mass	0.6 m_N
Nucleon Landau mass	0.83 m_N
incompressibility	240 MeV
Asymmetry energy	32.5 MeV

Parameters to fit normal nuclear matter



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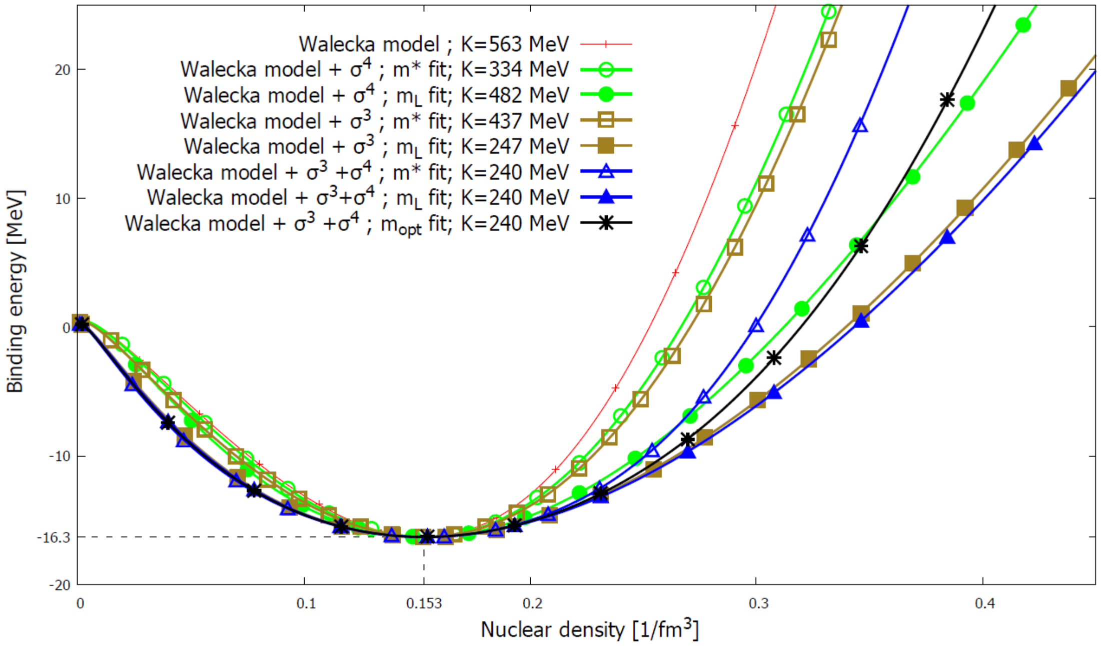
Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\,\frac{\partial p}{\partial n}$$

Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

Parameters to fit normal nuclear matter



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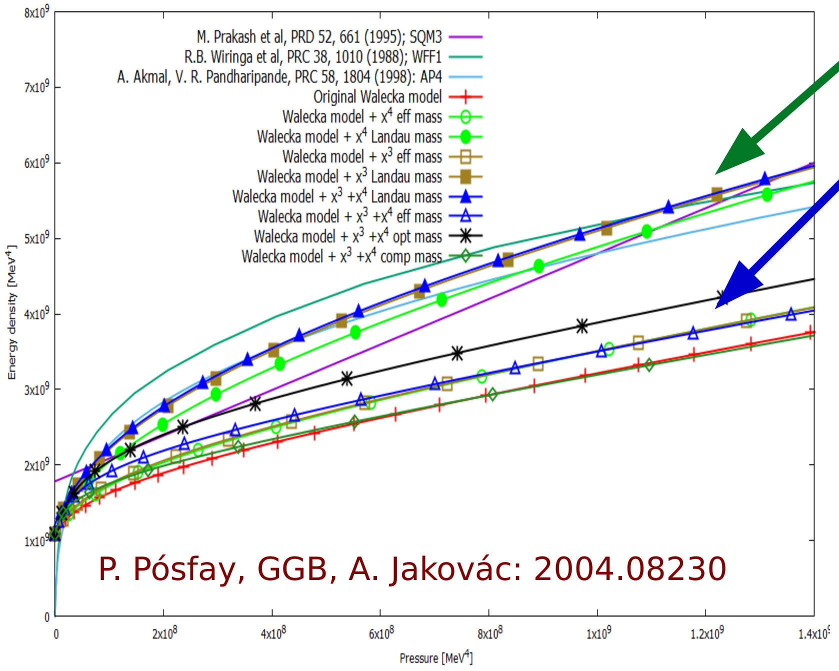
Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\frac{\partial p}{\partial n}$$

The effective mass and Landau mass are NOT independent!
The can not be fitted simultaneously

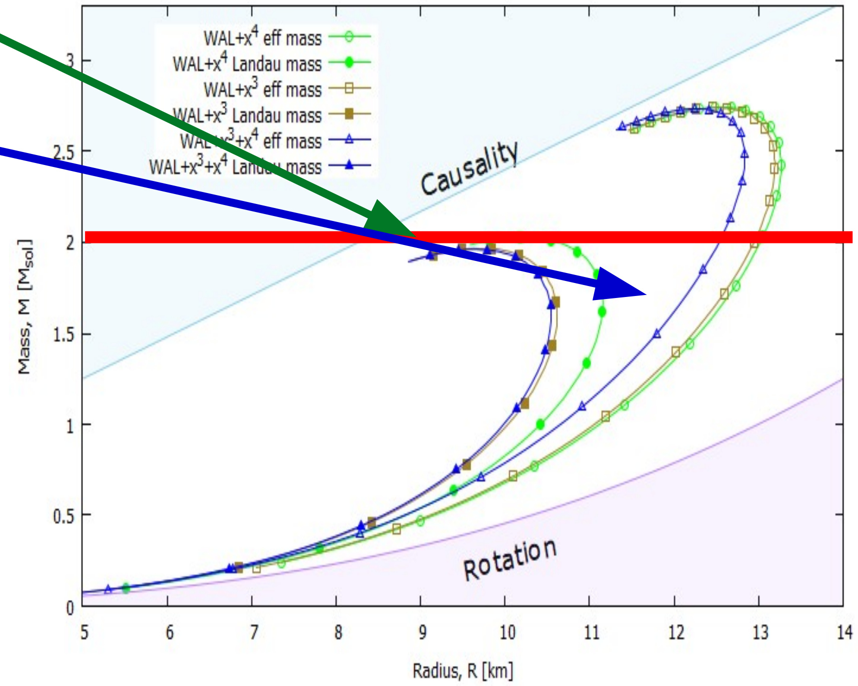
Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

The EoS & M-R of different model fits



Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

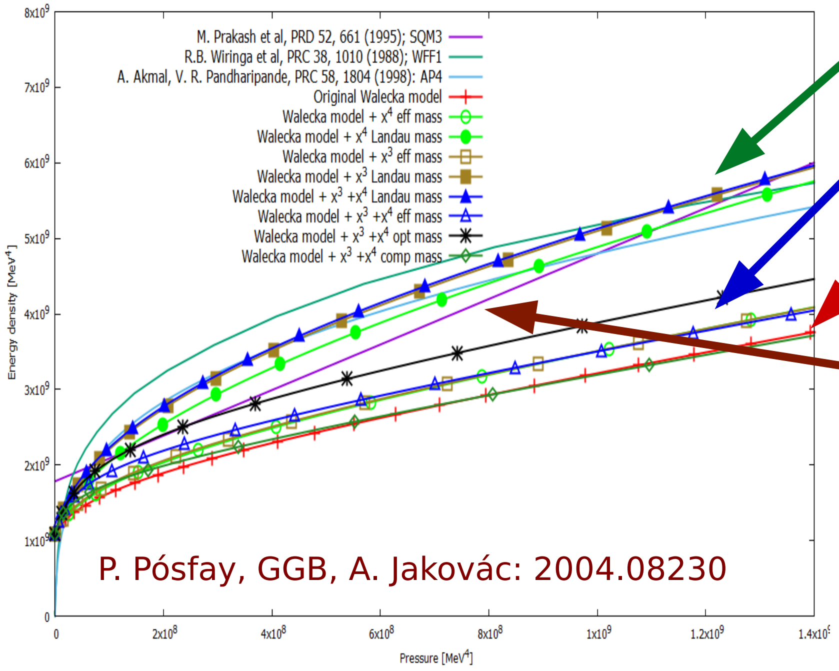


M-R diagram with these nuclear matter EoS

Cases with extra x^3 and/or x^4 terms provide similar band structures

→ Landau mass fits provide lower M_{max} closer to the observations

The EoS & M-R of different model fits



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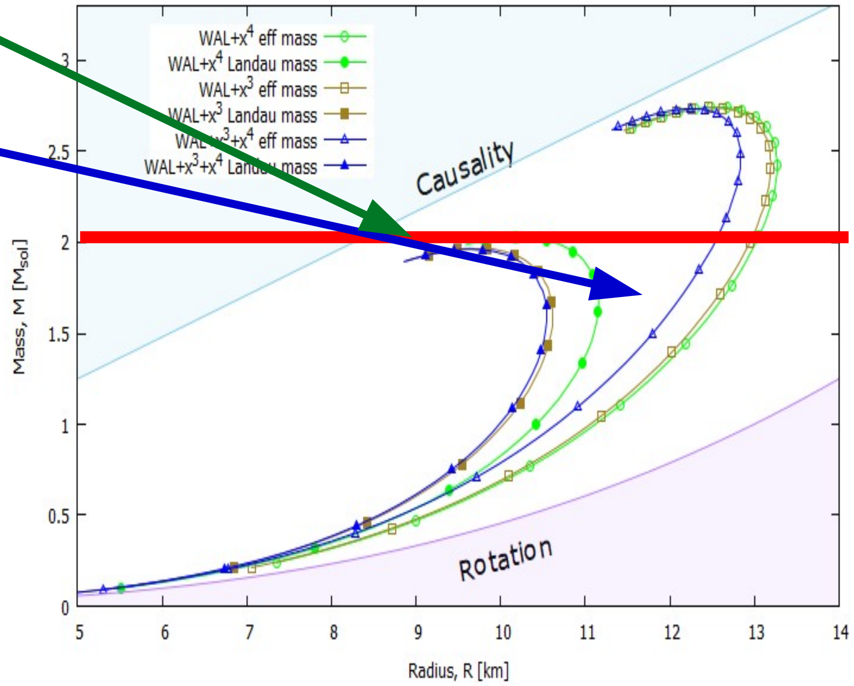
Original Walecka model

Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass



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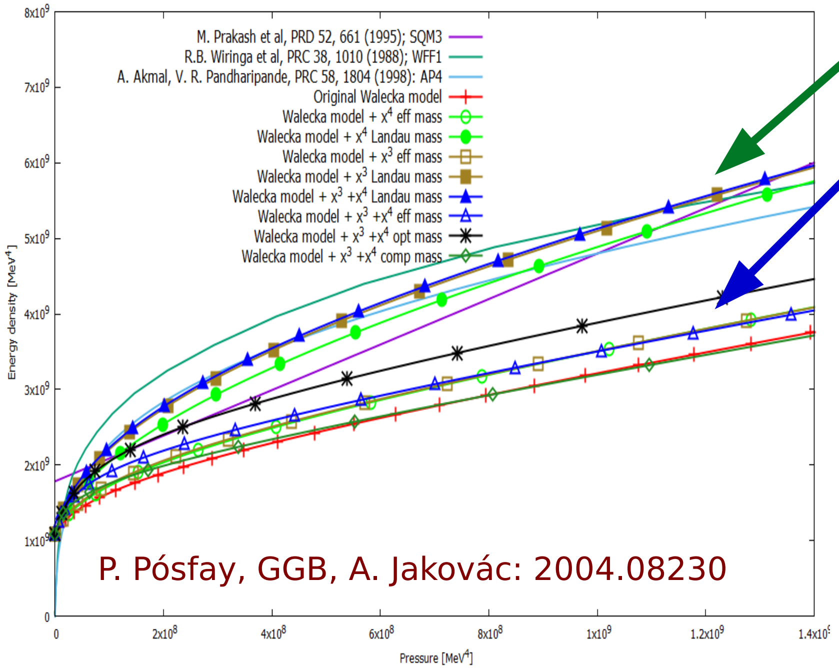

P. Pósfay, GGB, A. Jakovác: 2004.08230

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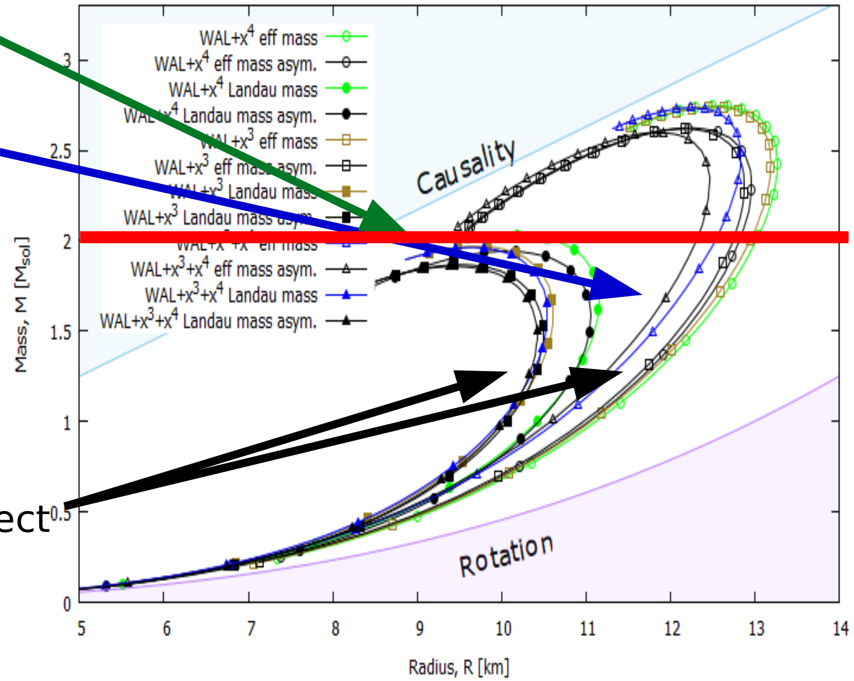


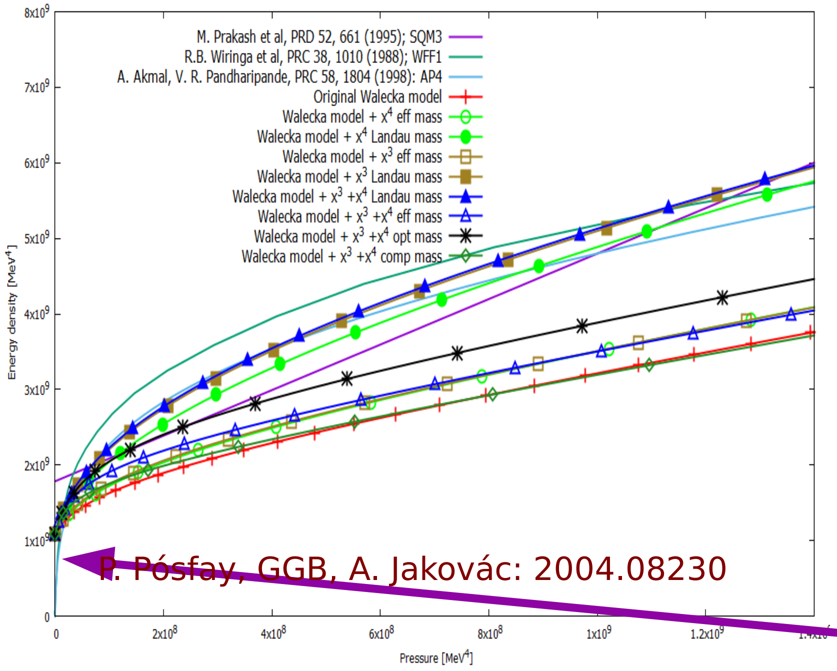
M-R diagram with these nuclear matter EoS

Cases with extra x^3 and/or x^4 terms provide similar band structures

→ Landau mass fits provide lower M_{max} closer to the observations

→ Nuclear ASYMMETRY result in 10-20% lower M_{max}

The EoS & M-R of different model fits



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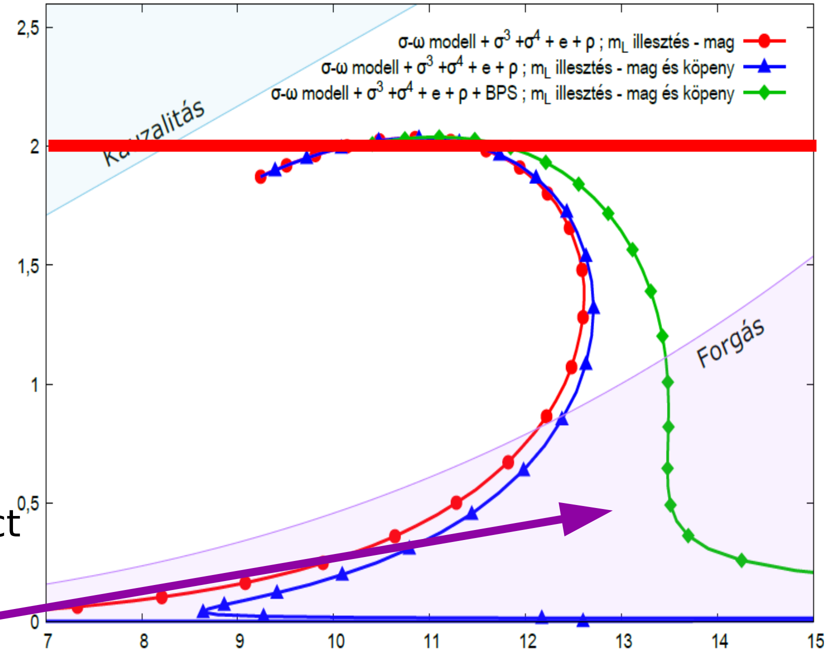
Realistic nuclear EoS: WFF1, AP4 (SQM) support the Landau mass

Assymetry (electrons) is weak effect

Crust (BPS) make more realistic



M-R diagram with these nuclear matter EoS

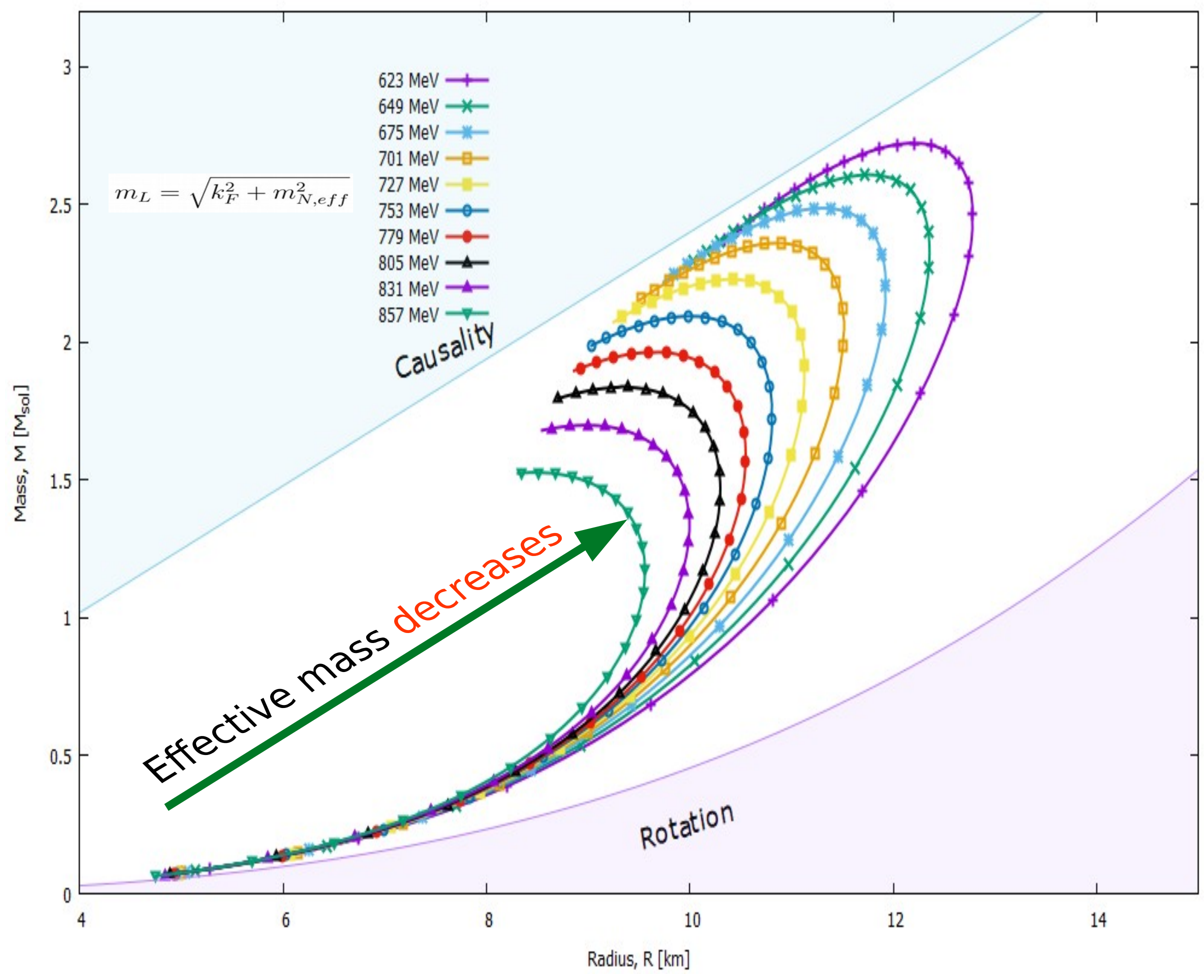
Cases with extra x^3 and/or x^4 terms provide similar band structures

→ Landau mass fits provide lower M_{max} closer to the observations

→ Nuclear ASYMMETRY result in 10-20% lower M_{max}

→ Adding CORE with BPS has no effect on M_{max}, only on R (~km)

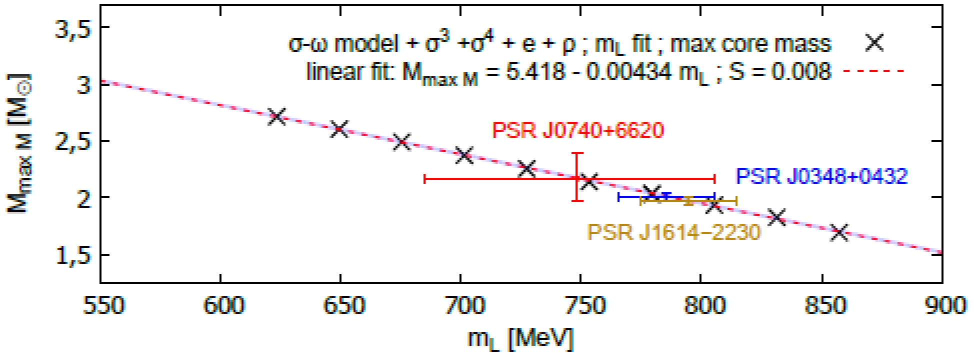
The M-R diagrams: EoS & Landau mass fit



Evolution/scaling in M_{max} appears

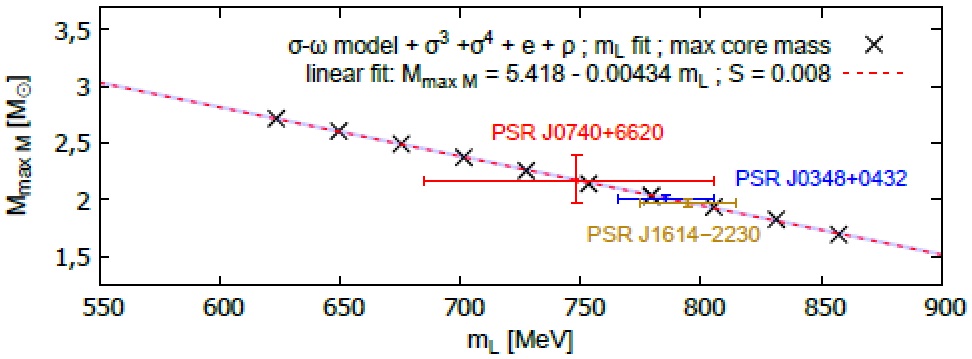
- The M_{max} is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters



P. Pósfay, GGB, A. Jakovác: 2004.08230

Scaling: maximum star mass vs. nuclear parameters

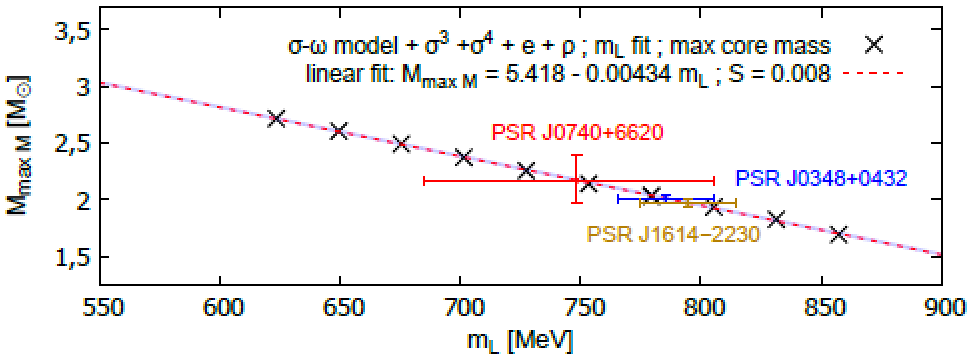


Maximal mass
with Landau mass

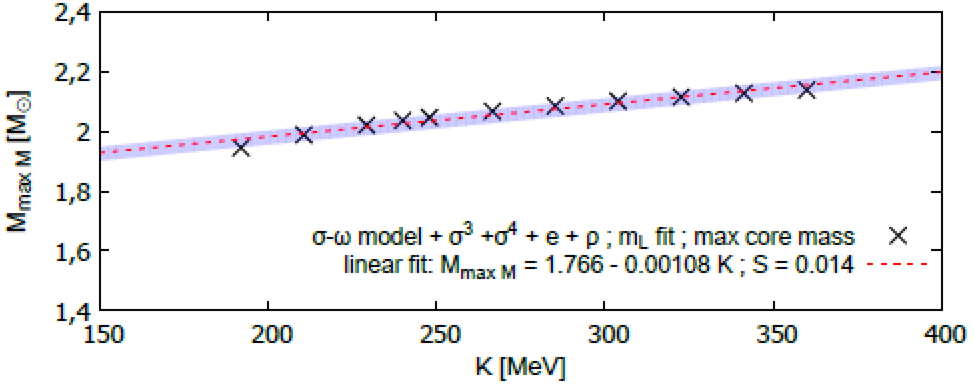
$$M_{maxM}(m_L)[\mathrm{M}_\odot] = 5.418 - 0.00434\, m_L[\mathrm{MeV}]$$

P. Pósfay, GGB, A. Jakovác: 2004.08230

Scaling: maximum star mass vs. nuclear parameters



a)



Maximal mass

with Landau mass

$$M_{maxM}(m_L)[\mathrm{M}_\odot] = 5.418 - 0.00434\, m_L\,[\mathrm{MeV}]$$

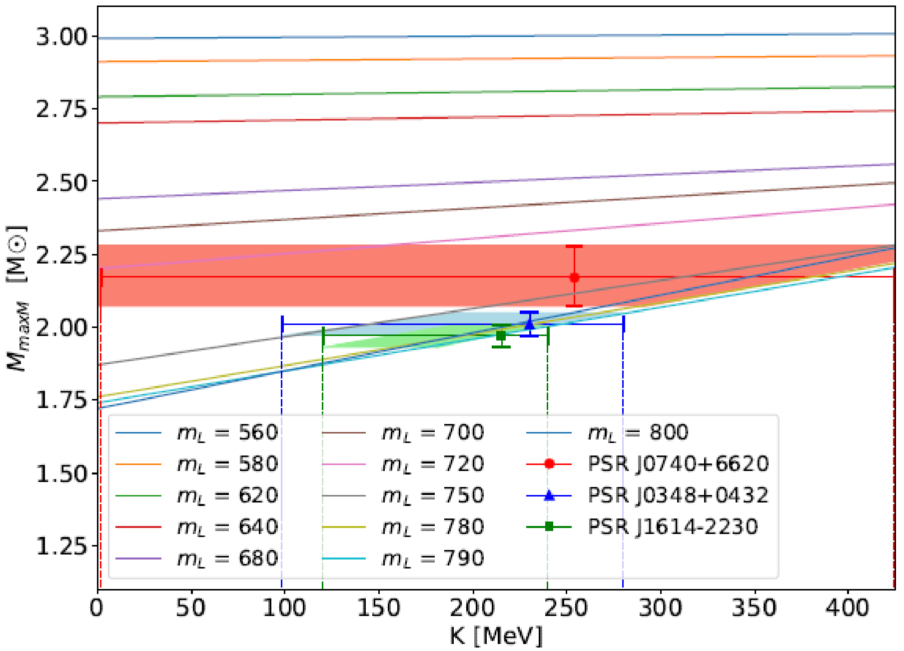
with (in)compressibility

$$M_{maxM}(K)[\mathrm{M}_\odot] = 1.766 + 0.00110K\ [\mathrm{MeV}]$$

$$\Delta M_{max}(\delta m_L) \overset{10\times}{>} \Delta M_{max}(\delta K)$$

P. Pósfay, GGB, A. Jakovác: 2004.08230

Scaling: maximum star mass vs. nuclear parameters



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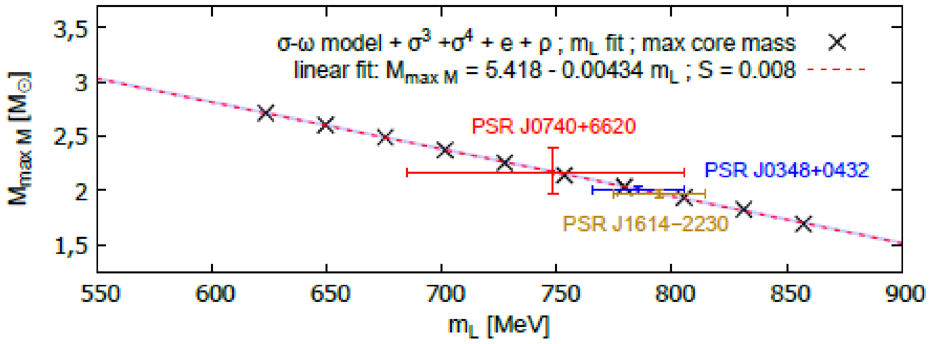
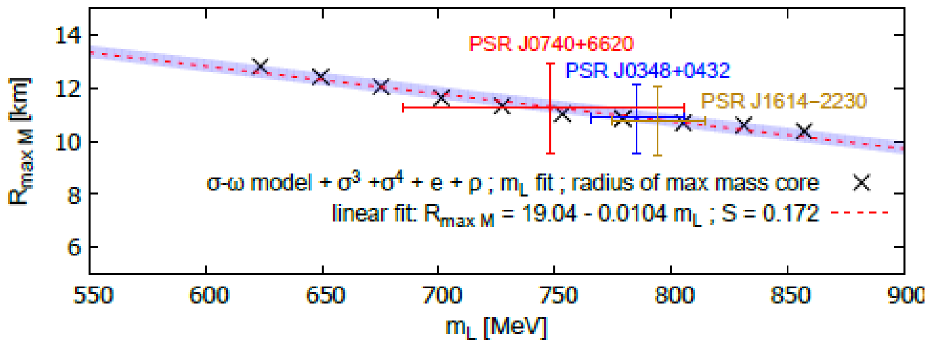
Combine these to a 2-parameter fit:

$$M_{maxM}(m_L, K)[\mathrm{M}_\odot] = 6.29 - 0.00574\, m_L\ [\mathrm{MeV}] - 0.00379\, K\ [\mathrm{MeV}] + 0.00000524\, m_L \cdot K\ [\mathrm{MeV}^2]\,.$$

GGB, P. Pósfay, B. Szigeti, Jakovác, EPJ ST 229 (2020) 3605

Scaling: maximum star mass vs. nuclear parameters





Maximal mass & its radius with Landau mass

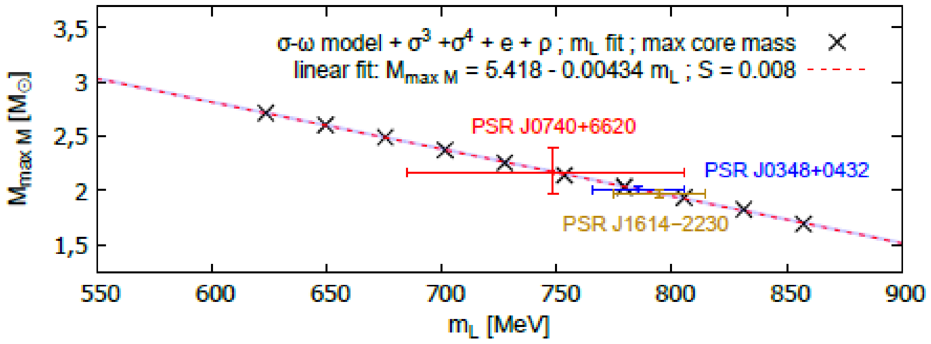
$$M_{maxM}(m_L)[\mathrm{M}_\odot] = 5.418 - 0.00434\, m_L\,[\mathrm{MeV}]$$
$$R_{maxM}(m_L)[\mathrm{km}] = 19.04 - 0.01040\, m_L\,[\mathrm{MeV}]$$

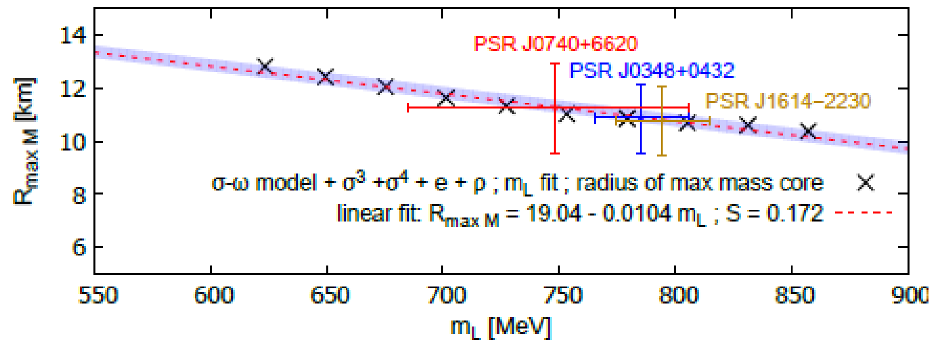
with (in)compressibility

$$M_{maxM}(K)[\mathrm{M}_\odot] = 1.766 + 0.00110K\ [\mathrm{MeV}]$$
$$R_{maxM}(K)[\mathrm{km}] = 8.878 + 0.00767K\ [\mathrm{MeV}]$$

GGB, P. Pósfay, B. Szigeti, Jakovác, EPJ ST 229 (2020) 3605

Scaling: maximum star mass vs. nuclear parameters





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$$R_{maxM}(K)[\mathrm{km}] = 8.878 + 0.00767K\ [\mathrm{MeV}]$$

Calculation for maximal mass star

Measured: $M_{maxM} \rightarrow (m_L\ \&\ K) \rightarrow R_{maxM}$

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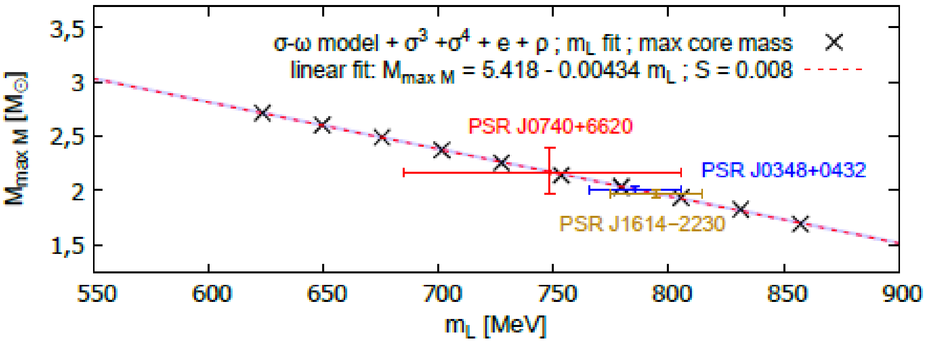
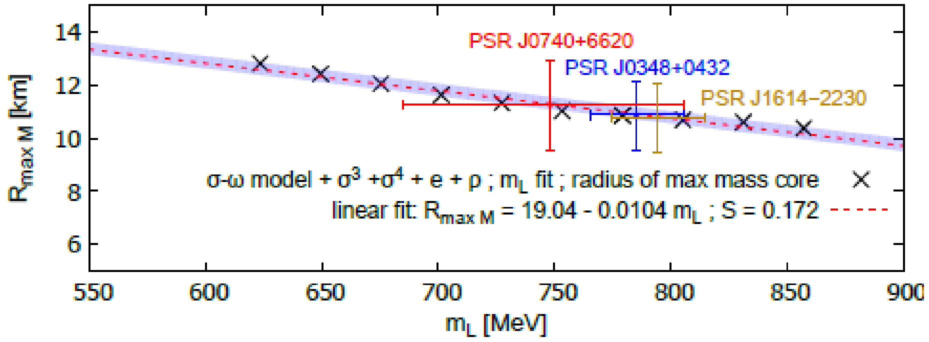
Scaling: maximum star mass vs. nuclear parameters





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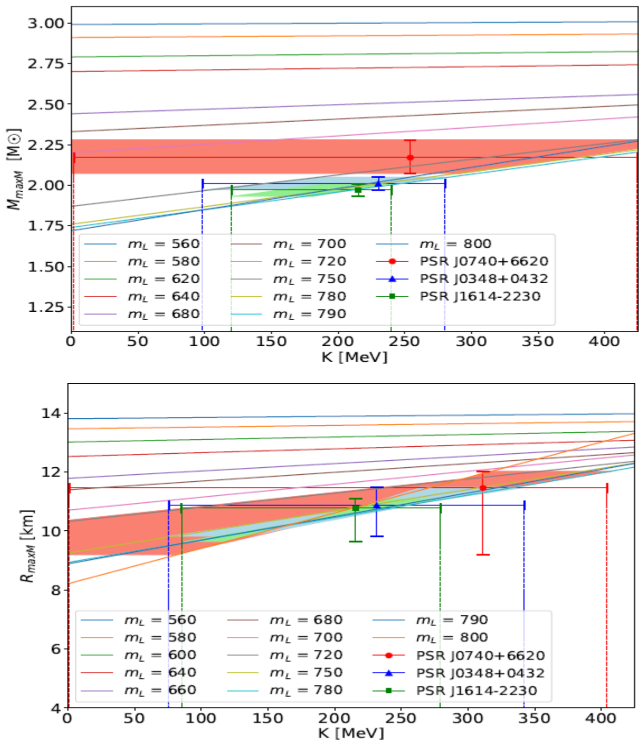
Calculation for maximal mass star

Pulsar	R_{maxM} [km]	M_{maxM} [$\mathrm{M}_\odot$]	m_L [MeV]	K [MeV]
PSR J0740+6620	$11.25^{+1.06}_{-1.04}$	$2.17^{+0.11}_{-0.10}$ *	$748.39^{+63.3}_{-57.2}$	$351.8^{+115}_{-84.5}$
PSR J0348+0432	$10.87^{+0.82}_{-0.80}$	$2.01^{+0.04}_{-0.04}$ *	$785.25^{+20.0}_{-20.3}$	$206.4^{+42.7}_{-20.5}$
PSR J1614−2230	$10.77^{+0.82}_{-0.80}$	$1.97^{+0.04}_{-0.04}$ *	$794.47^{+20.1}_{-20.4}$	$170.0^{+15.5}_{-20.9}$

GGB, P. Pósfay, B. Szigeti, Jakovác, EPJ ST 229 (2020) 3605

Scaling: maximum star mass vs. nuclear parameters



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$$R_{maxM}(m_L, K)[\mathrm{km}] = 27.51 - 0.0239\, m_L\ [\mathrm{MeV}] - 0.0241\, K\ [\mathrm{MeV}] + 0.0000411\, m_L \cdot K\ [\mathrm{MeV}^2]$$

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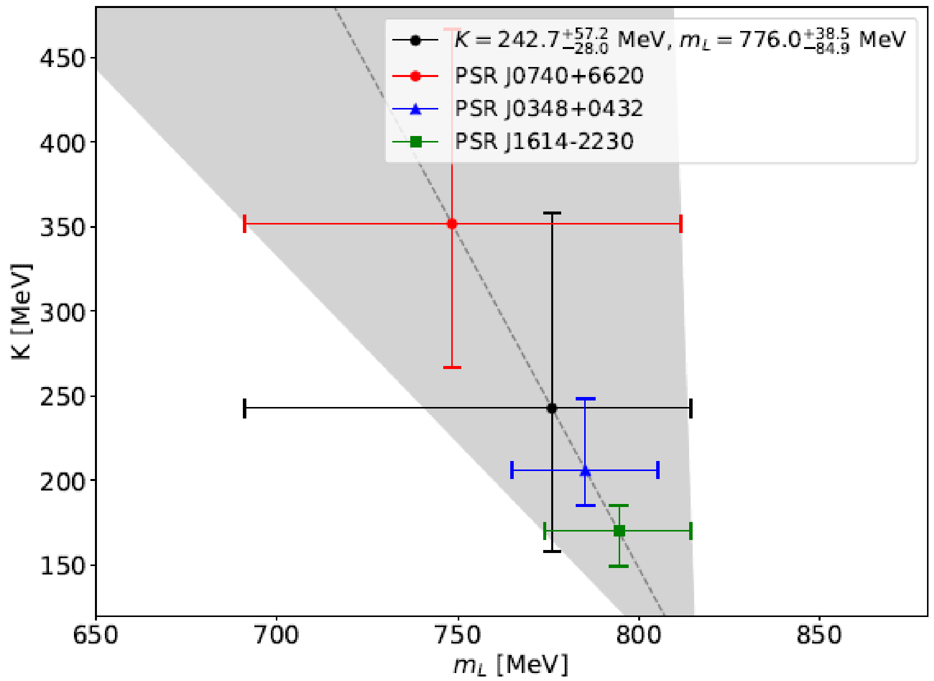
From data: Maximum star mass vs. nuclear parameters



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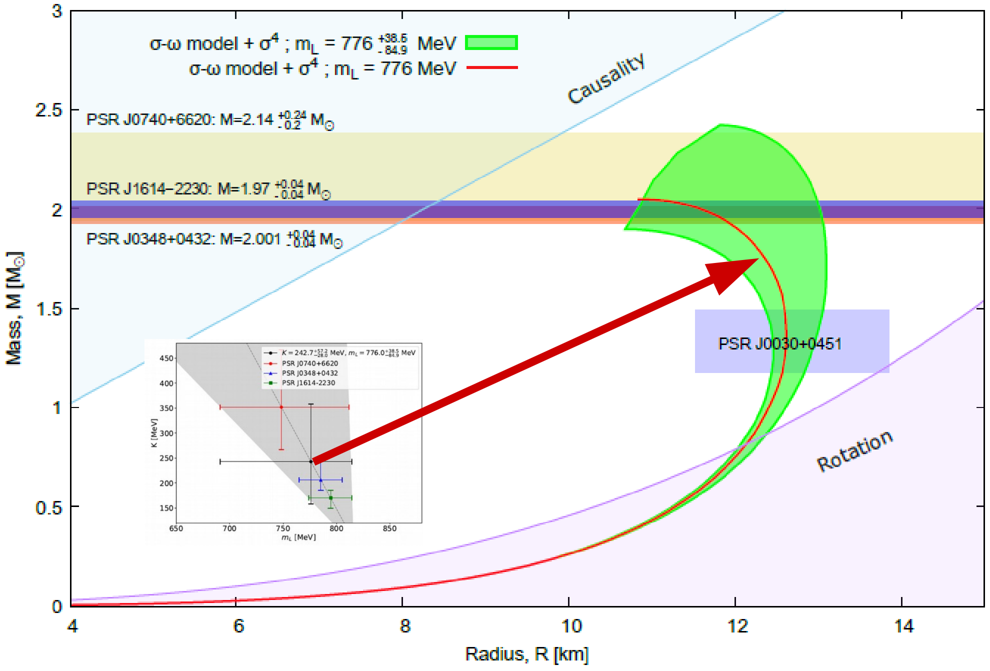

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Results from data using fit formulae:

$$m_L = 776.0^{+38.5}_{-84.9}\ \mathrm{MeV\ and}\ K = 242.7^{+57.2}_{-28.0}\ \mathrm{MeV}$$

Explore the uncertainties…

… using the brute force

D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay: Phys. Part & Nucl. 51 (2020) 725, + H. Grigorian EPJ ST 229 (2020) 3615

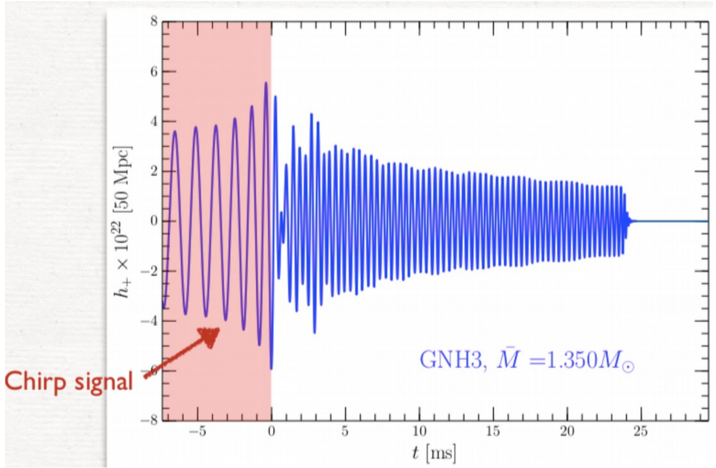
Love Number & tidal deformability

Extension of a standard TOV solver (i.e. numerically an integration of coupled ODEs):

Ansatz for the metric including a l=2 perturbation

$$\begin{aligned} ds^2 &= -e^{2\Phi(r)}\left[1+H(r)Y_{20}(\theta,\varphi)\right]dt^2 \\ &\quad +e^{2\Lambda(r)}\left[1-H(r)Y_{20}(\theta,\varphi)\right]dr^2 \\ &\quad +r^2\left[1-K(r)Y_{20}(\theta,\varphi)\right]\left(d\theta^2+\sin^2\theta d\varphi^2\right) \end{aligned}$$

Following Hinderer et al. 201



Integrate standard TOV system:

$$\begin{aligned} e^{2\Lambda} &= \left(1-\frac{2m_r}{r}\right)^{-1}, \\ \frac{d\Phi}{dr} &= -\frac{1}{\epsilon+p}\frac{dp}{dr}, \\ \frac{dp}{dr} &= -(\epsilon+p)\frac{m_r+4\pi r^3 p}{r(r-2m_r)}, \\ \frac{dm_r}{dr} &= 4\pi r^2\epsilon. \end{aligned}$$

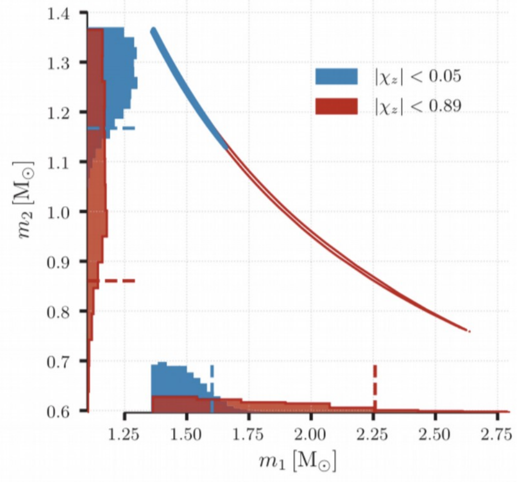
EoS to be provided ε(p)

And additional eqs. for perturbations:

$$\frac{dH}{dr} = \beta \qquad (11)$$

$$\begin{aligned} \frac{d\beta}{dr} &= 2\left(1-2\frac{m_r}{r}\right)^{-1} H\left\{-2\pi\left[5\epsilon+9p+f(\epsilon+p)\right]\right. \\ &\quad \left. +\frac{3}{r^2}+2\left(1-2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2}+4\pi r p\right)^2\right\} \\ &\quad +\frac{2\beta}{r}\left(1-2\frac{m_r}{r}\right)^{-1}\left\{-1+\frac{m_r}{r}+2\pi r^2(\epsilon-p)\right\}. \end{aligned}$$

(K(r) given by H(r))



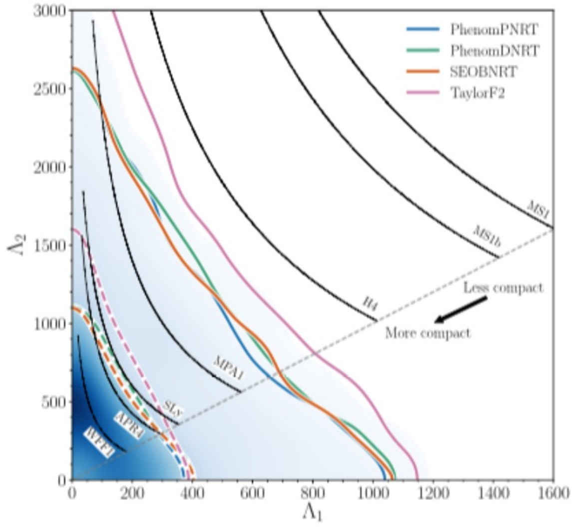
Love Number & tidal deformability

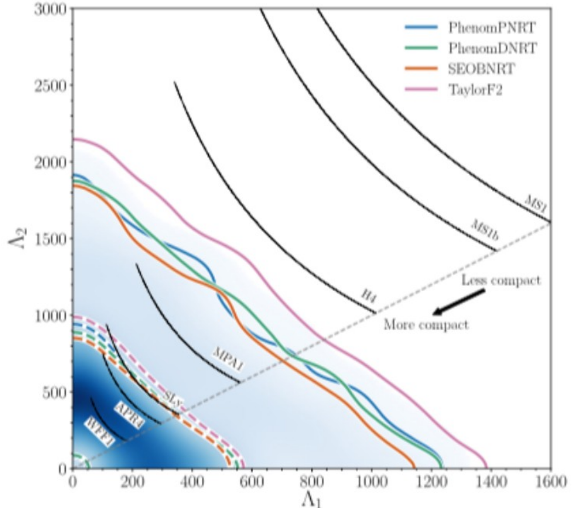
$$y = \frac{R\beta(R)}{H(R)}$$

$$\Lambda = \frac{2}{3}\frac{R^5}{M^5}k_2$$

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \times \Big\{2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)] + 3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C)\Big\}^{-1}$$

where $C = M/R$ is the compactness of the star.





Brute force: Bayesian analysis

Data: $\vec{\pi}_q = \{m_{L(i)}, K_{0(j)}, S_{0(k)}\}$

Likelihood for given independent constraints:

$$P(E\,|\vec{\pi}_q) = \prod_w P(E_w\,|\vec{\pi}_q)$$

Posterior:

$$P(\vec{\pi}_q|E) = \frac{P(E\,|\vec{\pi}_q)\,P(\vec{\pi}_q)}{\sum_{p=0}^{N-1} P(E\,|\vec{\pi}_p)\,P(\vec{\pi}_p)}$$

Marginalization (1-parameter):

$$P\left(m_{L(i)}\,|E\right) = \sum_{j,k} P\left(m_{L(i)}, K_{0(j)}, S_{0(k)}\,|E\right)$$

D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay: Phys. Part & Nucl. 51, 725 (2020)

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Likelihood for GW170817:

$$P(E_{GW}\,|\pi_q) = \int_l \beta(\Lambda_1(n_c), \Lambda_2(n_c))\,\mathrm{d}n_c$$

Likelihood for maximal mass

$$P(E_M\,|\pi_q) = \Phi(M_q, \mu_{\mathrm{C}}, \sigma_{\mathrm{C}}) \times \Phi(M_q, \mu_{\mathrm{A}}, \sigma_{\mathrm{A}}) \times \mathcal{N}(M_q, \mu_{\mathrm{U}}, \sigma_{\mathrm{U}})$$

Likelihood for mass & radius

$$P(E_{MR}\,|\pi_q) = 0.5 \int_l \mathcal{N}\left(\mu_M^{(1)}, \sigma_M^{(1)}, \mu_R^{(1)}, \sigma_R^{(1)}, \alpha^{(1)}\right)\mathrm{d}n_c + 0.5 \int_l \mathcal{N}\left(\mu_M^{(2)}, \sigma_M^{(2)}, \mu_R^{(2)}, \sigma_R^{(2)}, \alpha^{(2)}\right)\mathrm{d}n_c,$$

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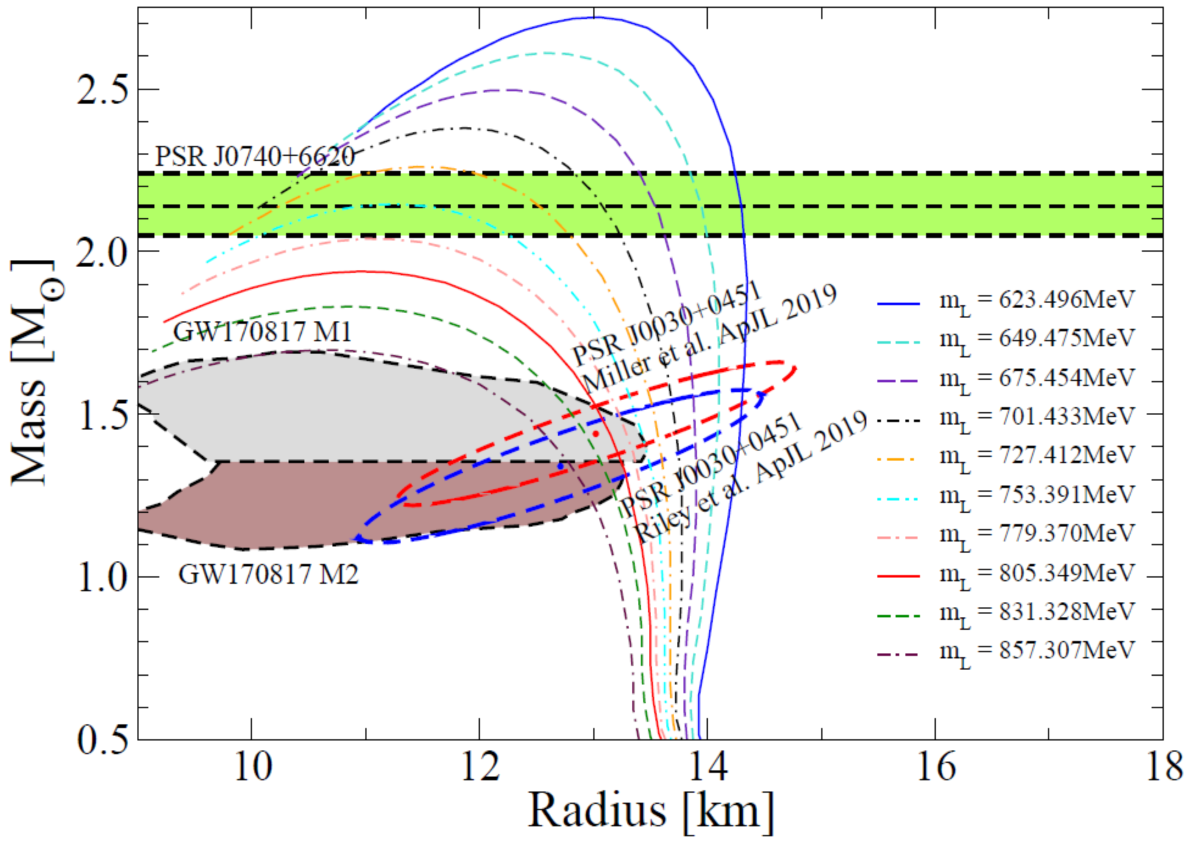
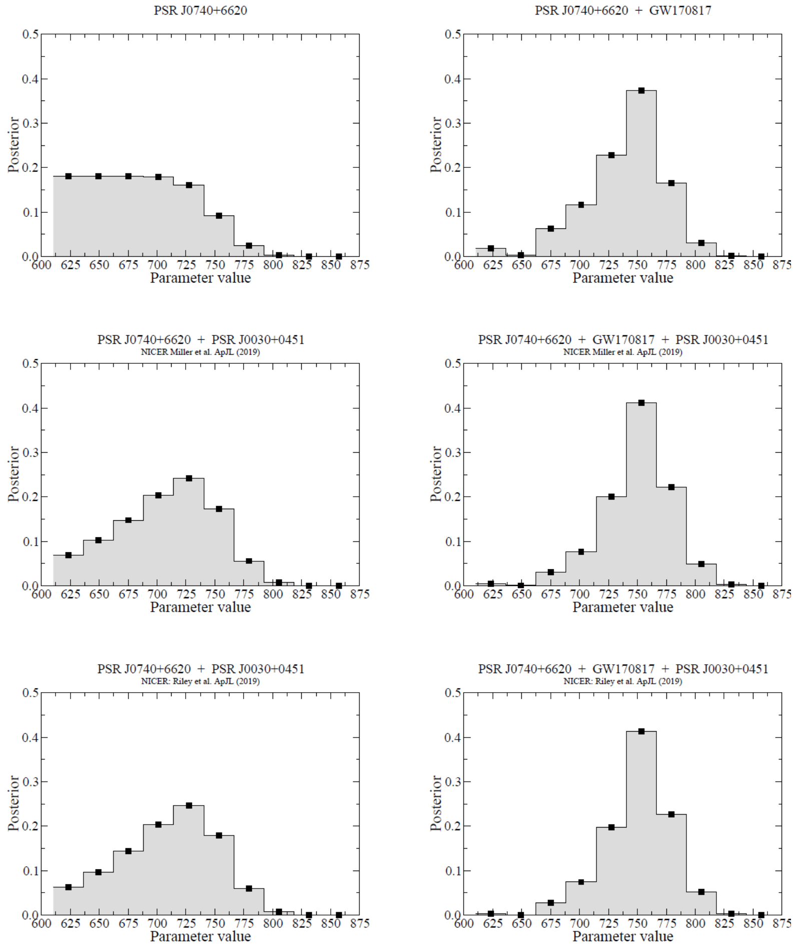
Brute force: Bayesian analysis

Data with m_L only





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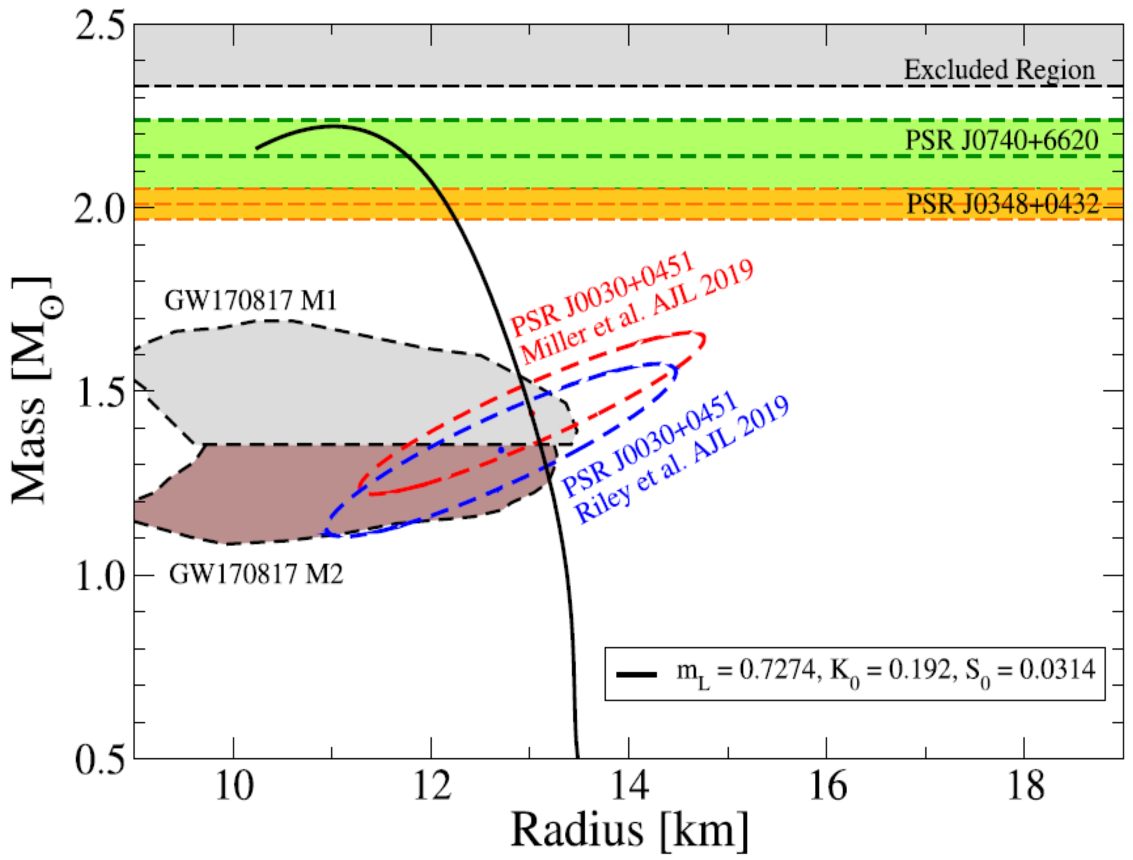

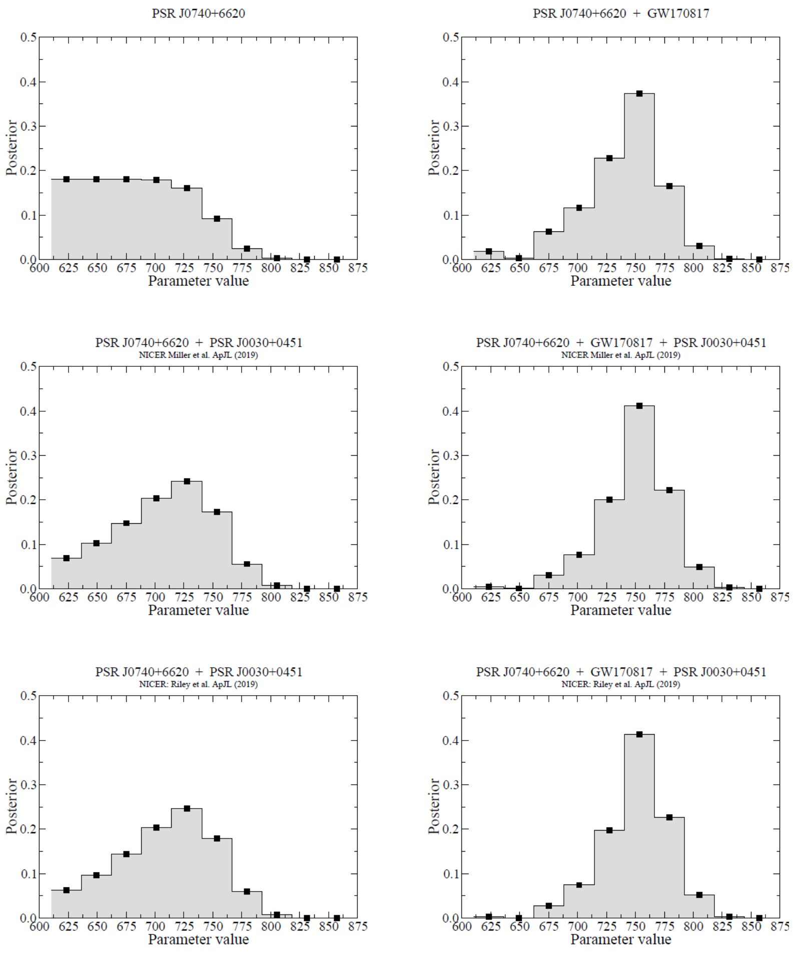

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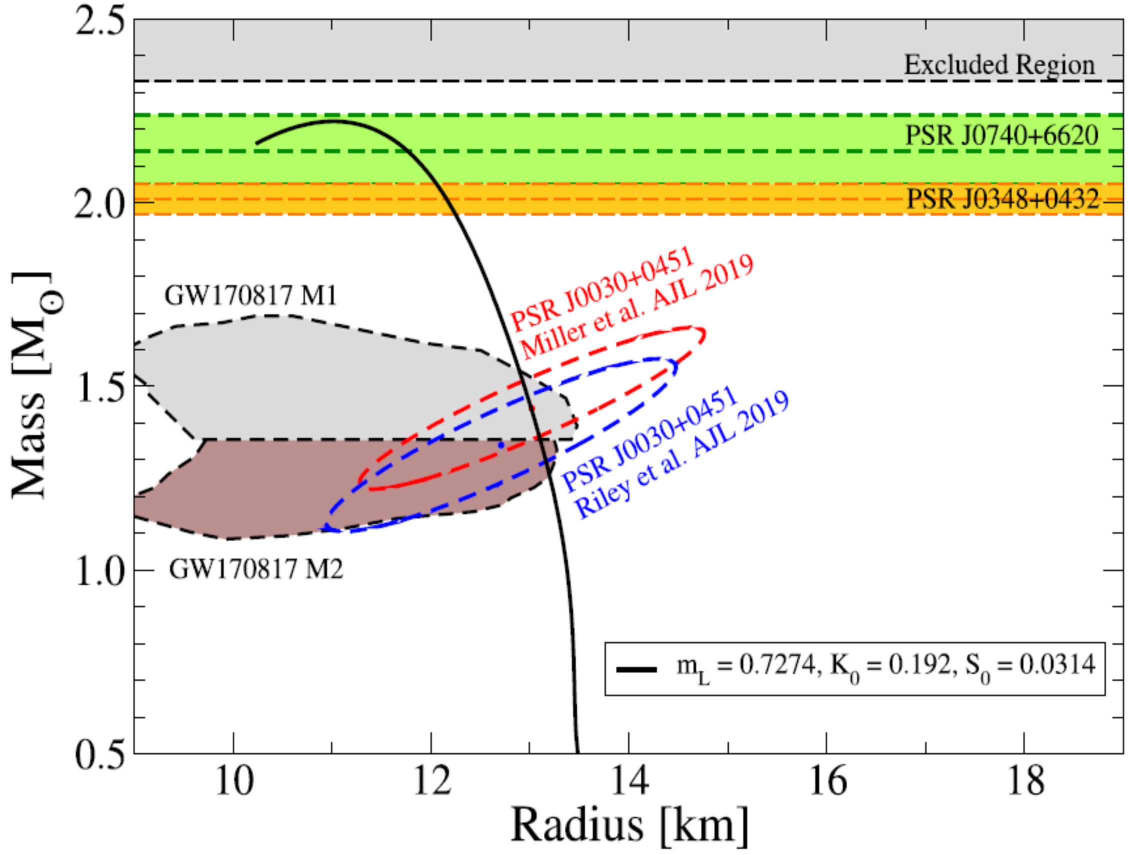
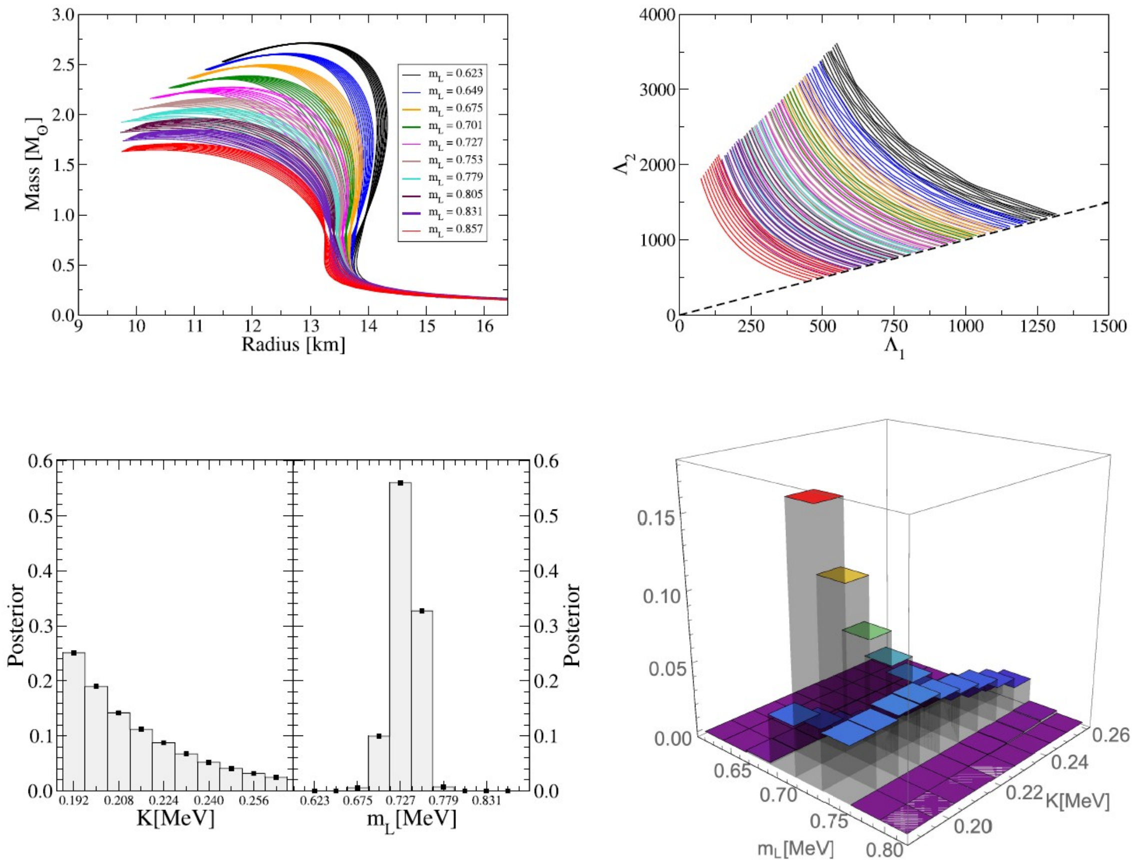
Brute force: Bayesian analysis

Data with m_L & K





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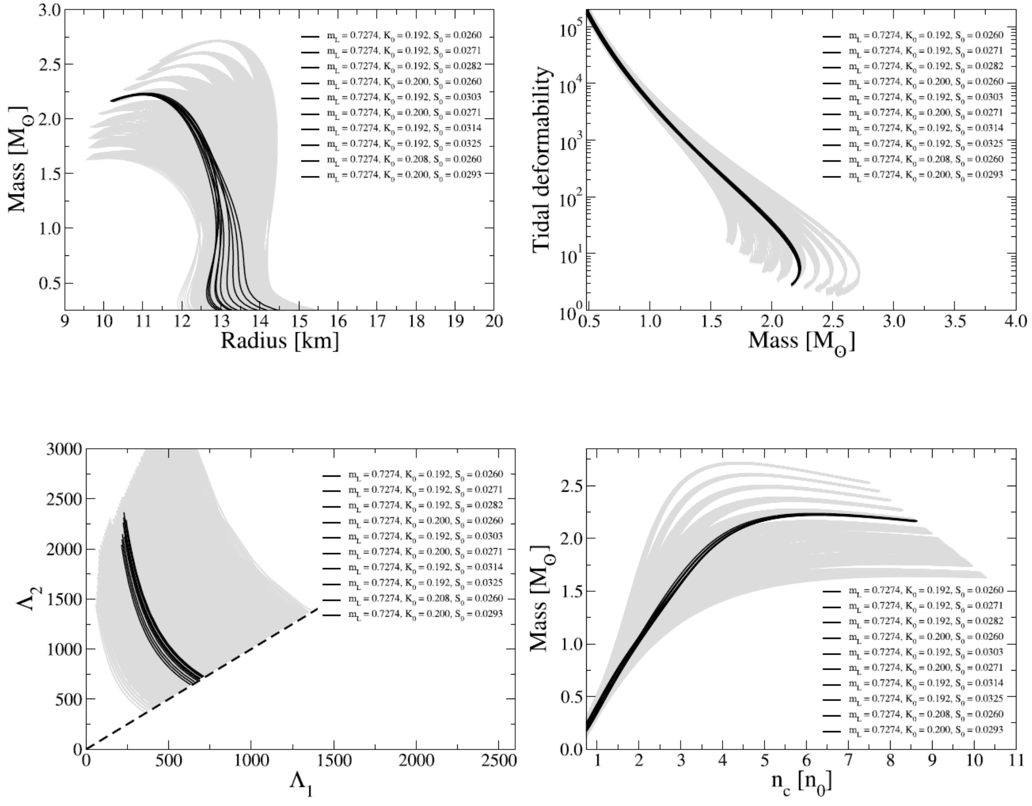
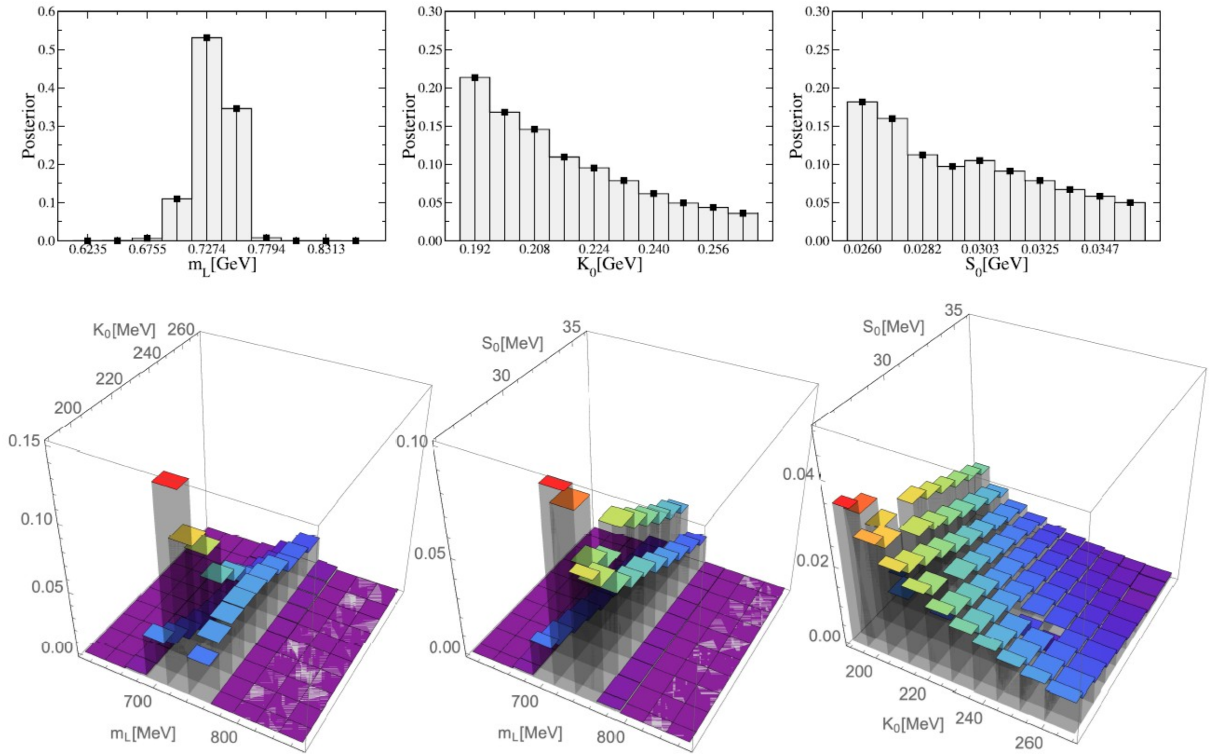
Brute force: a Bayesian analysis

Data with m_L & K & S_0





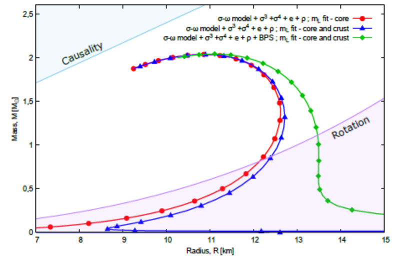
D. Alvarez-Castillo, A. Ayrian, GGB, P. Pósfay, H. Grigorian EPJ ST 229 (2020) 3615

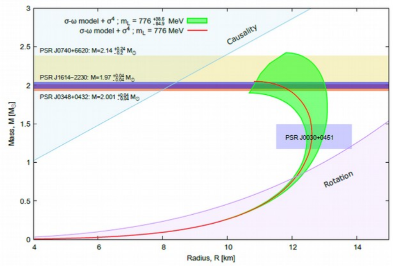
Conclusions: traditions vs brute force

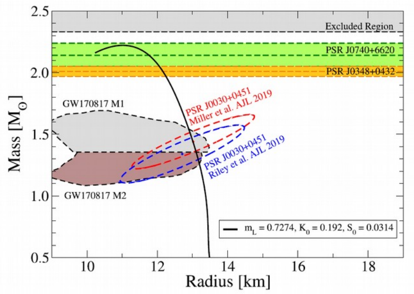
- **Traditional way: mean field model**
  - In CORE approximation: maximal mass provide a unique message:
    - → strong linear Landau mass dependence
    - → an order of magnitude smaller K-dependence

$$\Delta M_{max}(\delta m_L) \overset{10\times}{>} \Delta M_{max}(\delta K) \overset{10\times}{>} \Delta M_{max}(\delta a_{sym}).$$

  - Soft part of the EoS changes the CRUST, thus vary R
  - FRG: parameter 10-25% observables: 5-10%
- **Values & uncertainties – a cross check**
  - Traditional model: $m_L = 776.0^{+38.5}_{-84.9}$ MeV and $K = 242.7^{+57.2}_{-28.0}$ MeV
  - Bayesian model*: $m_L = 727.4 \pm 15$ MeV and $K = 232 \pm 20$ MeV







BACKUP

BREAKING NEWS

ALICE

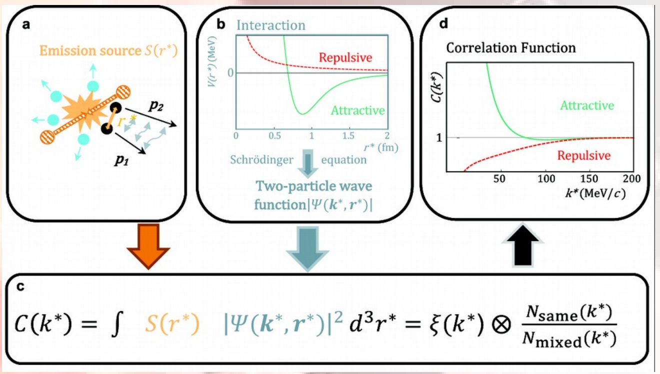
nature

Article | Published: 09 December 2020

Unveiling the strong interaction among hadrons at the LHC

ALICE Collaboration

Nature **588**, 232–238(2020) | Cite this article



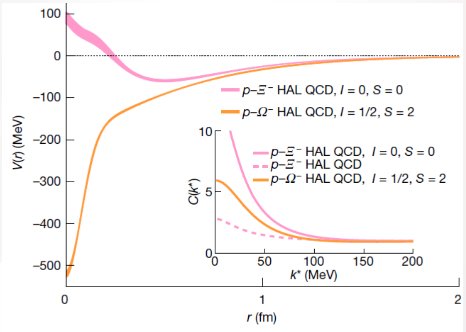


Fig. 4 | Potentials for the p–Ξ^- and p–Ω^- interactions. p–Ξ^- (pink) and p–Ω^-