Commonly used event classifiers for small systems and the associated biases

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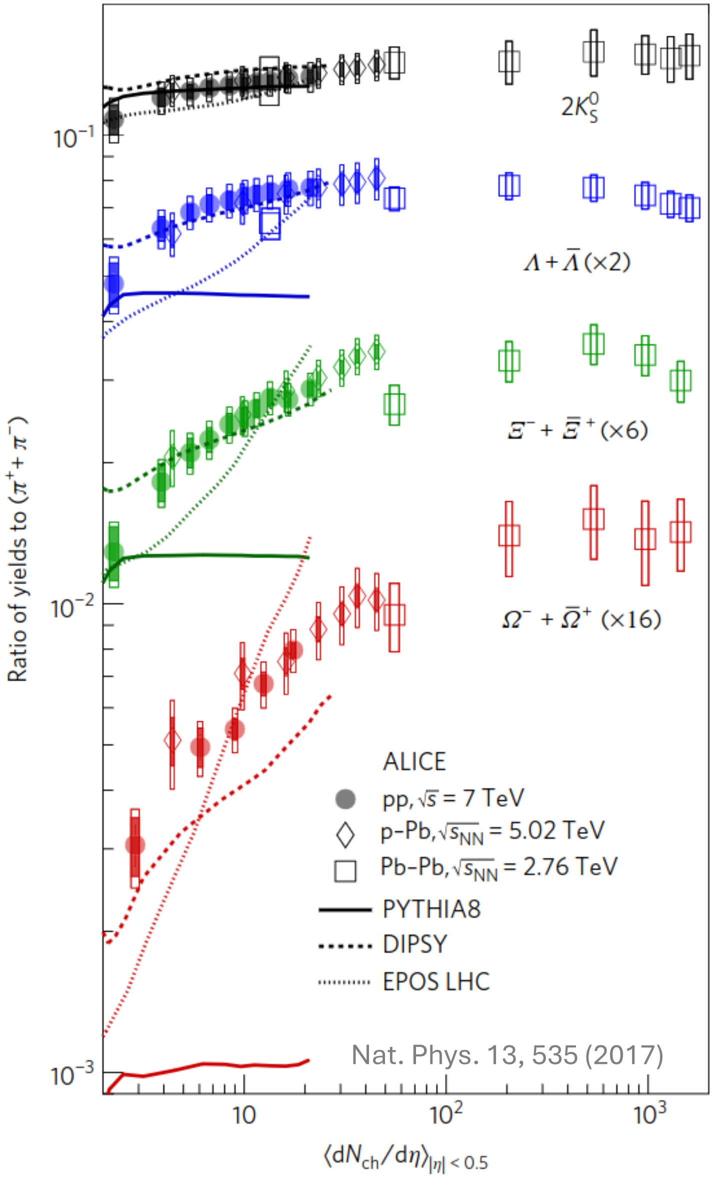
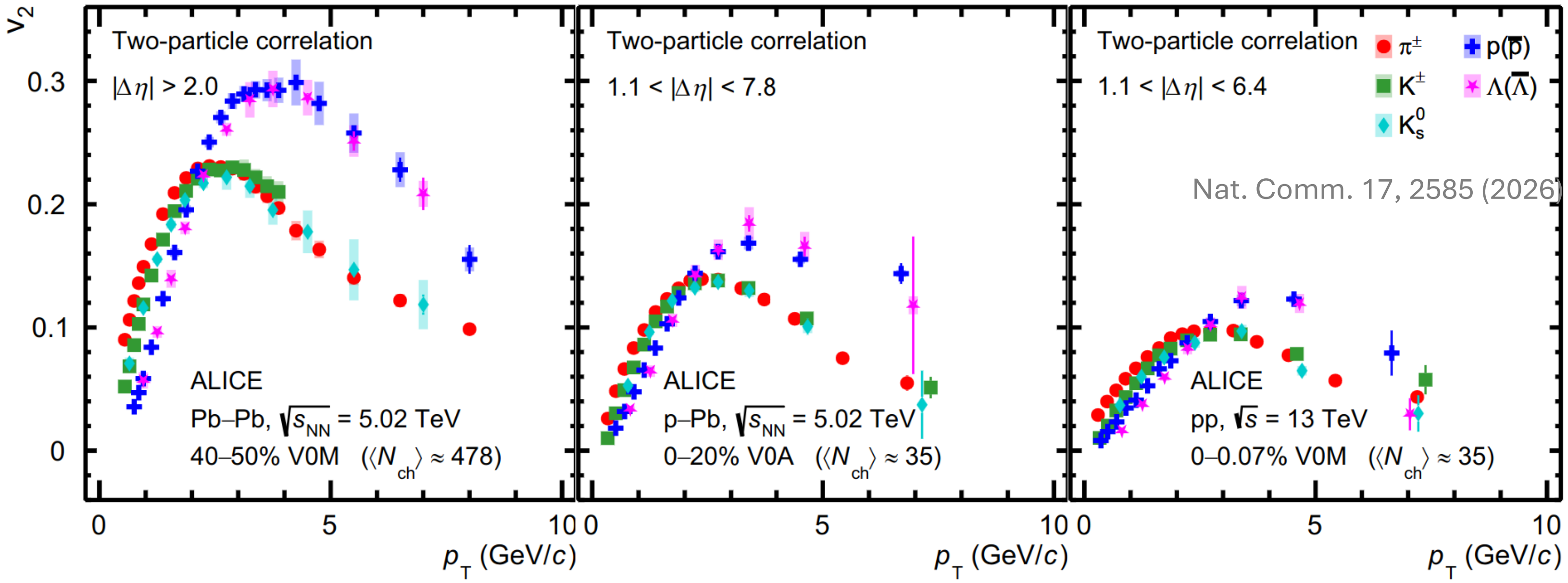
EMMI Workshop: QCD Challenges from pp to AA Collisions

Universita di Torino, July 6–10, 2026

QGP like Signals at the LHC



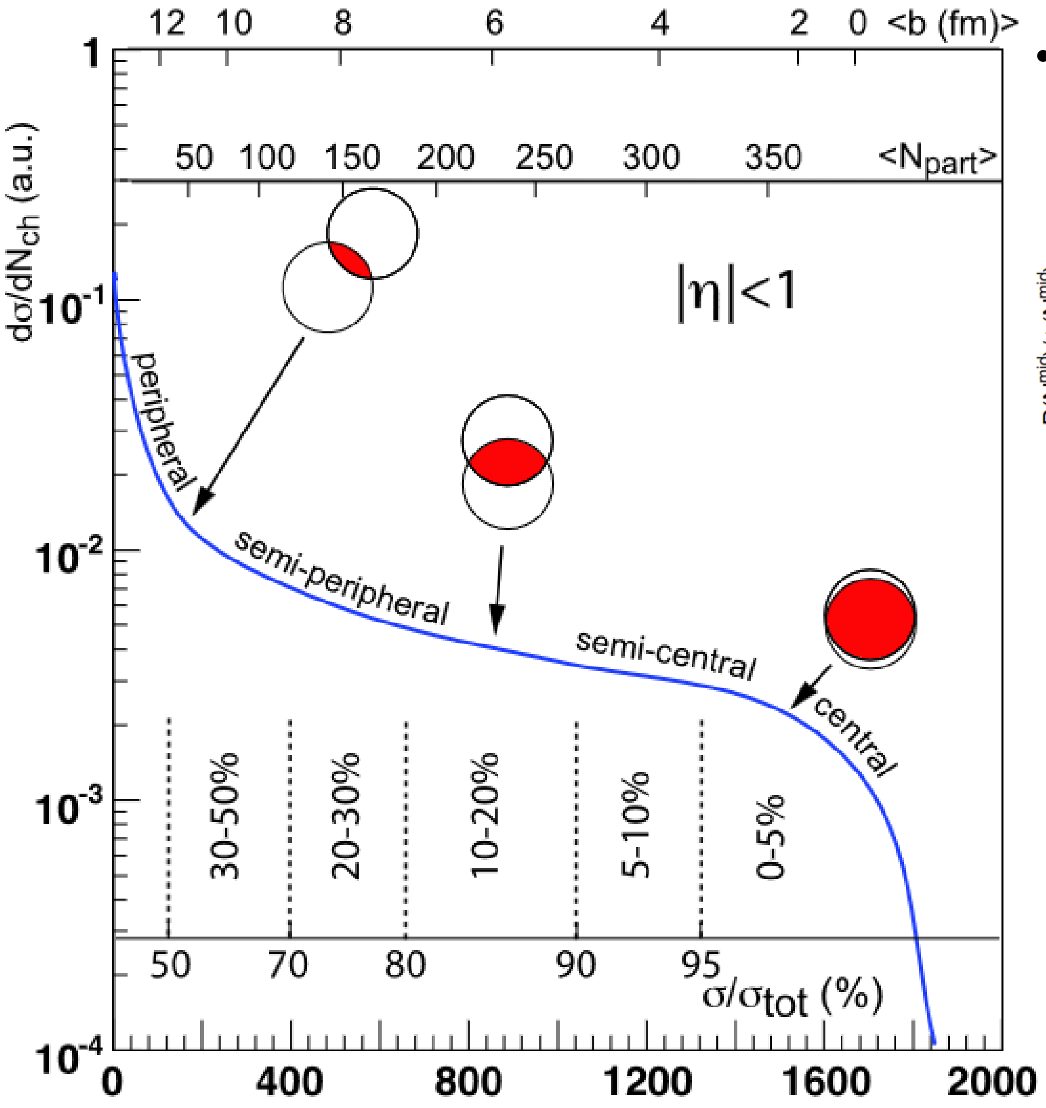


- Originally expected in heavy-ion collisions: now observed in pp
  - ➢ Long range angular correlations (ridge)
  - ➢ Strangeness enhancement with multiplicity
  - ➢ Non-zero v_2 and baryon meson separation
  - ➢ Radial flow like patterns in particle ratios

Central Question:

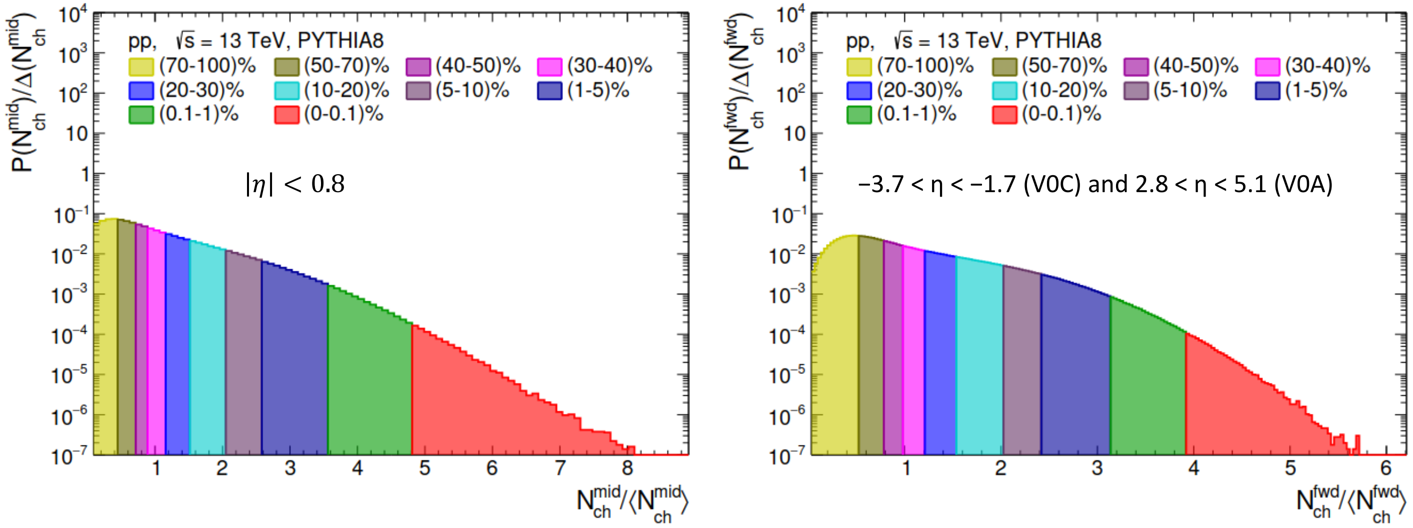
How much of this is real and how much depends upon how we select events?

In AA We Have Centrality — In pp We Don’t



Ann. Rev. Nucl. Part. Sci. 57, 205 (2007)

- Charged particle multiplicity in the final state is one of the most fundamental observable in collider experiments which serve as an estimator for centrality in AA collisions



- In AA collisions:
  - ✓ $N_{\mathrm{ch}} \rightarrow b$
- In pp or pA collisions:
  - ➢ N_{ch} fluctuates strongly

- Multiplicity driven by underlying event activity + Jet in small systems

S. Prasad, S. Tripathy, B. Sahoo, and R. Sahoo, Phys. Rept. 1181 (2026) 1-75

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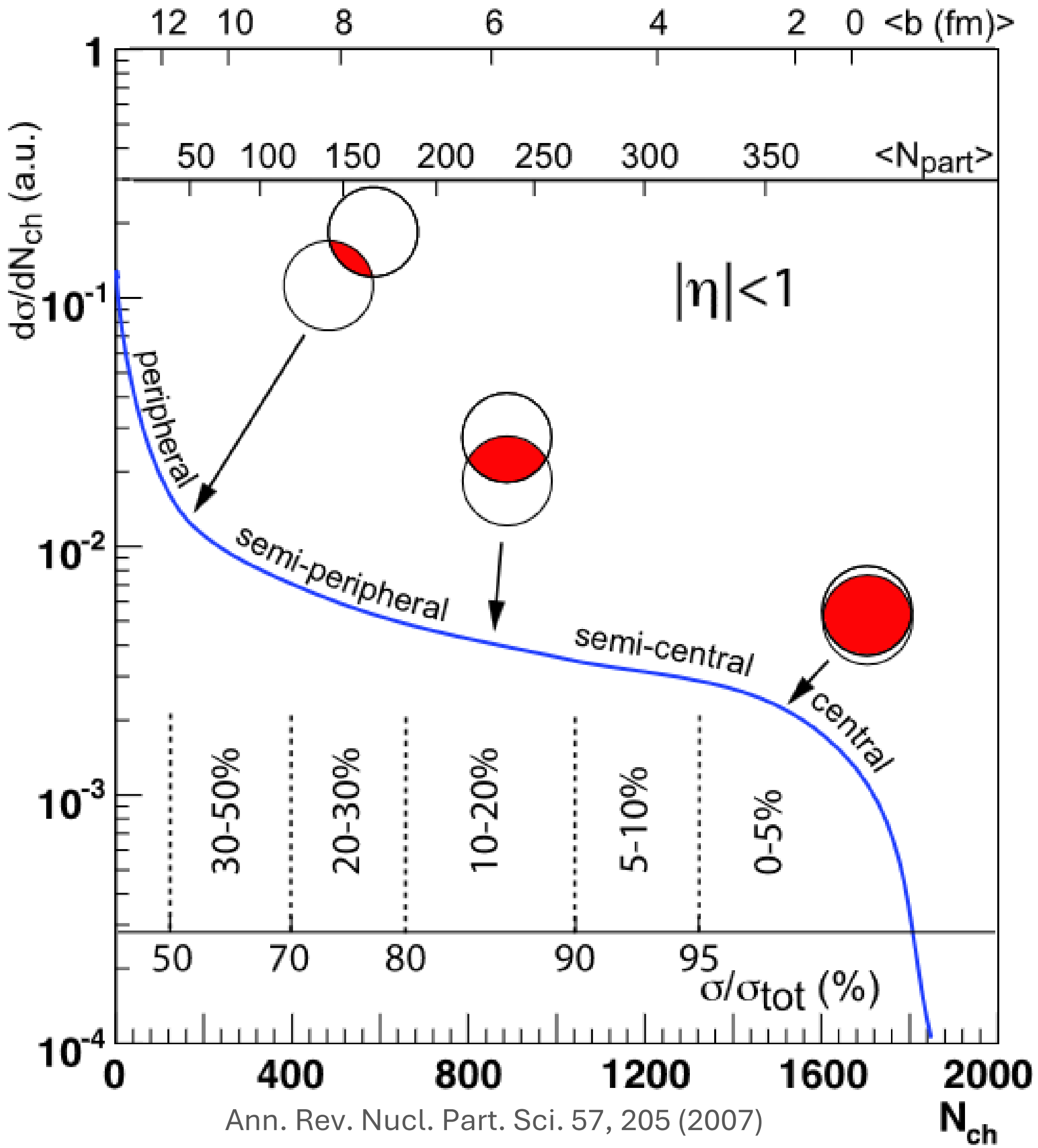

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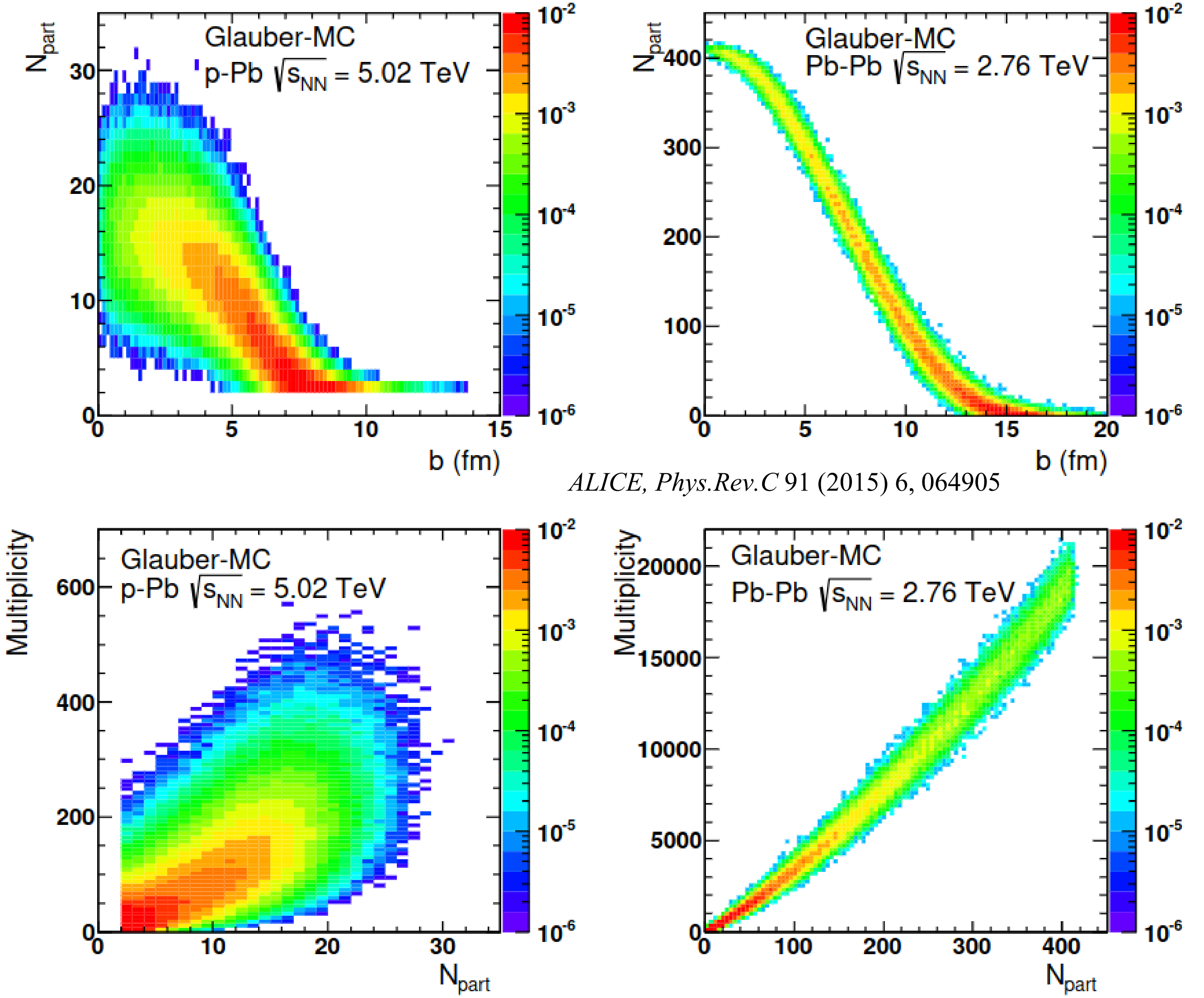
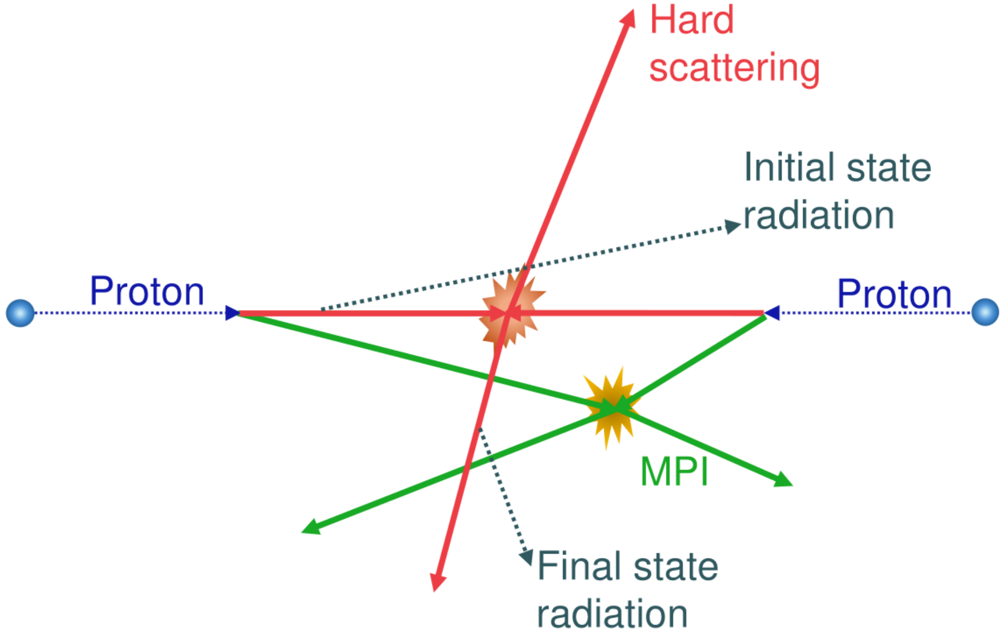


ALICE, Phys.Rev.C 91 (2015) 6, 064905

What Drives small systems? MPI as the Key Variable

- The primary interaction usually produces the highest-p_{T} process
- Additional parton-parton scatterings are called Multiple Parton Interactions (MPI)
- MPI are a major source of the Underlying Event (UE)
- They become increasingly important at LHC energies due to the large parton densities at small Bjorken-x

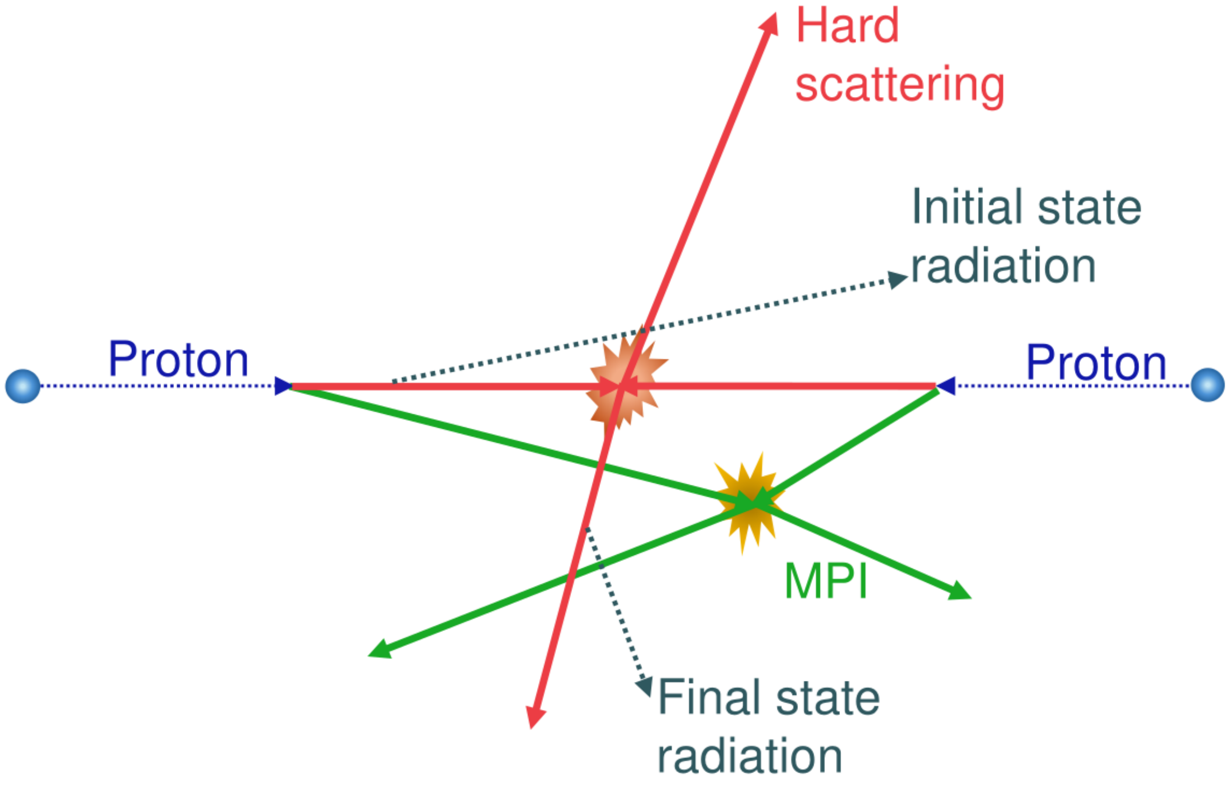


S. Prasad, S. Tripathy, B. Sahoo, and R. Sahoo, Phys. Rept. 1181 (2026) 1-75

	Central	Peripheral
Heavy-ion	• Small b • Large overlap region • Many participating nucleons (N_{part}) and binary nucleon-nucleon collisions (N_{coll}) • High particle multiplicity	• Large b • Small overlap region • Fewer N_{part} and N_{coll} • Lower multiplicity
proton-proton	Small pp impact parameter • Large overlap of parton densities • Higher probability of MPI • Higher event activity	Large pp impact parameter • Smaller overlap • Fewer MPIs • Lower event activity

What Drives pp Events? MPI as the Key Variable

- From MC studies we understood: MPI controls the UE activity
- We can not measure N_{mpi} in experiments



A good estimator should:

- Correlate strongly with N_{mpi}
- Not be contaminated by the hardest scattering
- Not share particles with the measurement

Available tools:

- Multiplicity-based: $N_{\mathrm{ch}}^{\mathrm{mid}}$, $N_{\mathrm{ch}}^{\mathrm{fwd}}$
- Event-shape: Transverse Spherocity (S_0), Transverse sphericity (S_{T}), flattenicity (ρ_{ch})
- UE-based:R_{T}

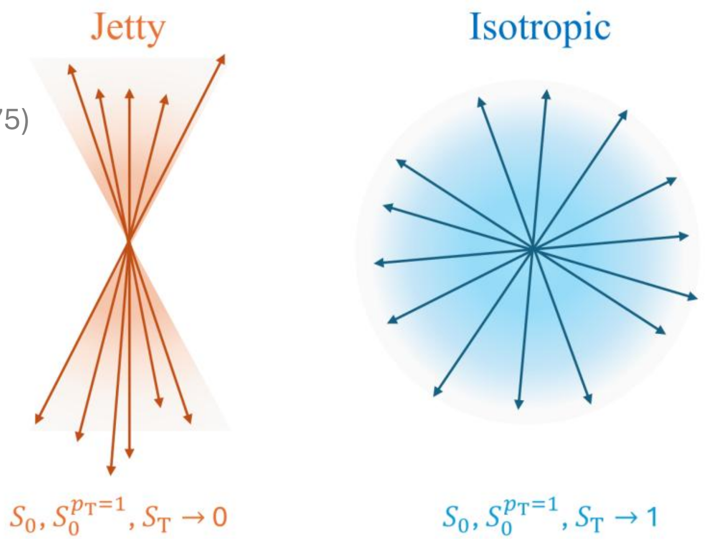
S. Prasad, S. Tripathy, B. Sahoo, and R. Sahoo, Phys. Rept. 1181 (2026) 1-75

Transverse Sphericity

- Transverse sphericity can be defined as the eigen values ($\lambda_1 > \lambda_2$) of the matrix:

$$\mathbf{S}_{\mathbf{xy}}^{\mathbf{Q}} = \frac{1}{\sum_i p_{\mathrm{T}i}} \sum_i \begin{pmatrix} p_{\mathrm{x}}{}_i^2 & p_{\mathrm{x}i}\, p_{\mathrm{y}i} \\ p_{\mathrm{y}i}\, p_{\mathrm{x}i} & p_{\mathrm{y}}{}_i^2 \end{pmatrix} \; [\mathrm{GeV}/c]$$

G. Hanson et. al., Phys. Rev. Lett 35, 1609 (1975)

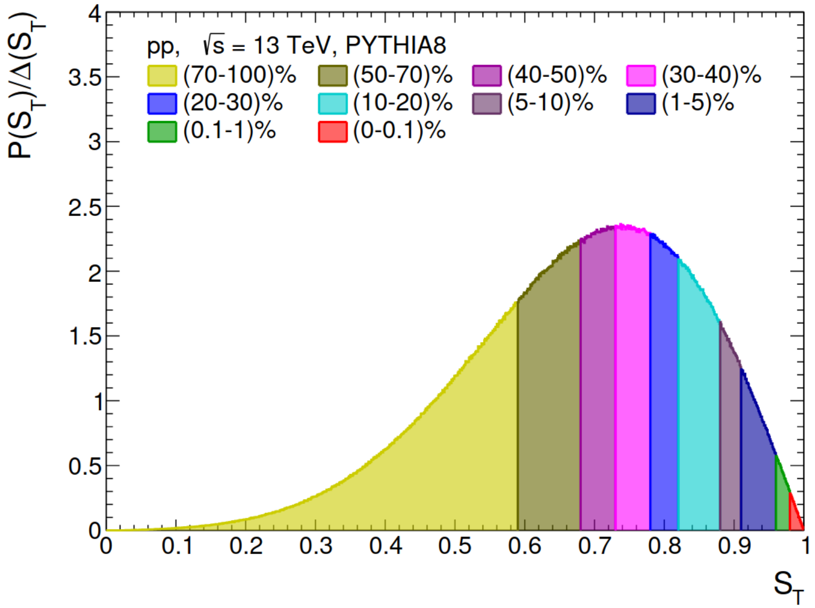


- Transverse sphericity calculated from above matrix is non-collinear
- Linearized matrix to calculate collinear safe sphericity:

$$\mathbf{S}_{\mathbf{xy}}^{\mathbf{L}} = \frac{1}{\sum_i p_{\mathrm{T}i}} \sum_i \frac{1}{p_{\mathrm{T}i}} \begin{pmatrix} p_{\mathrm{x}}{}_i^2 & p_{\mathrm{x}i}\, p_{\mathrm{y}i} \\ p_{\mathrm{y}i}\, p_{\mathrm{x}i} & p_{\mathrm{y}}{}_i^2 \end{pmatrix}$$

- Eigen values of $S_{\mathrm{xy}}^{\mathrm{L}}$ are λ_1, and λ_2 with $\lambda_1 > \lambda_2$
- Finally, $S_\mathrm{T} = \frac{2\lambda_2}{\lambda_1+\lambda_2}$
- Similar to spherocity, the extreme limits of sphericity:

$$S_\mathrm{T} = \begin{cases} 0 & \text{“pencil-like” limit} \\ 1 & \text{“isotropic” limit.} \end{cases}$$



S. Prasad, B. Sahoo, S. Tripathy, N. Mallick, R. Sahoo, Phys. Rev. C. 111, 044902 (2025)
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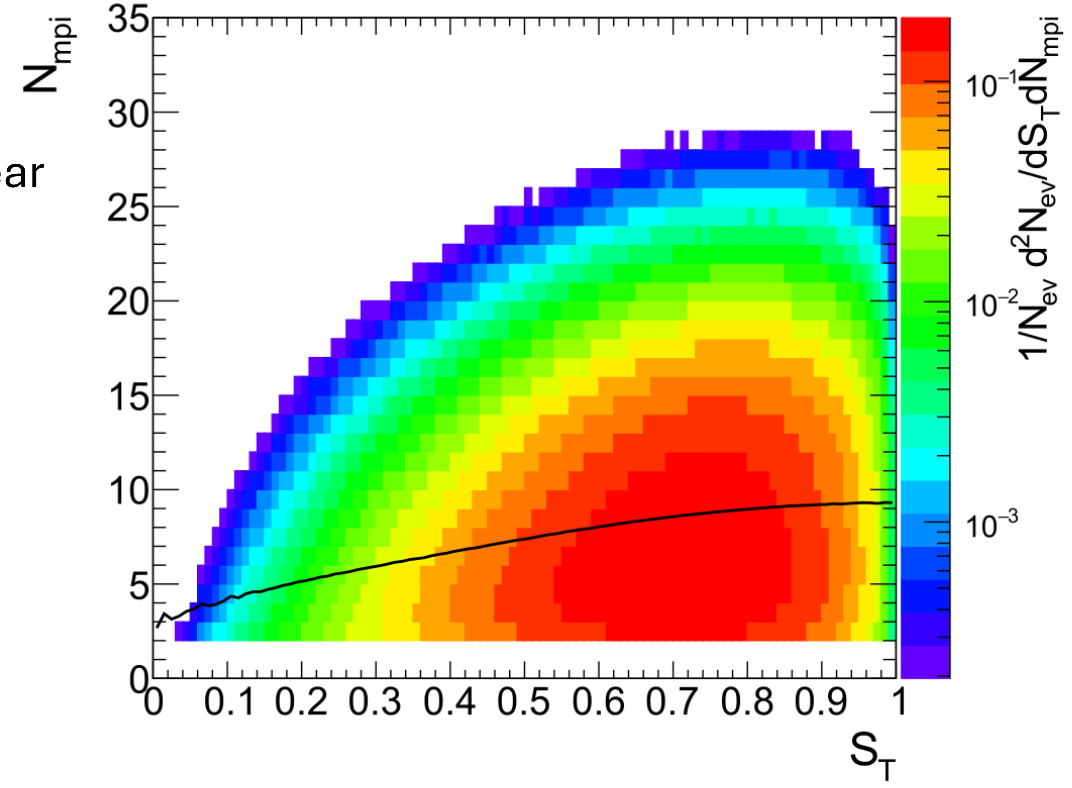

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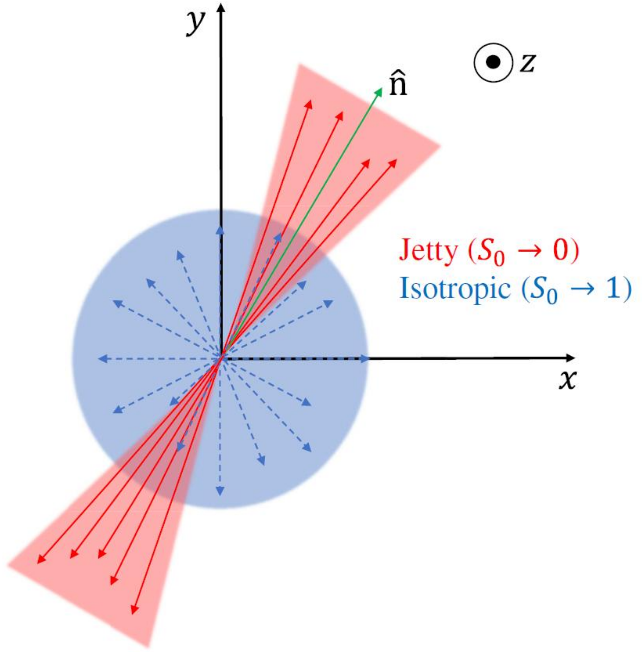
Transverse Spherocity

- Event shape observable: separates events based on geometrical shapes

$$S_0 = \frac{\pi^2}{4} \min_{\hat{n}} \left(\frac{\Sigma_i |\vec{p}_{T_i} \times \hat{n}|}{\Sigma_i |\vec{p}_{T_i}|} \right)^2$$

E. Cuautle et. al., arXiv:1404.2372

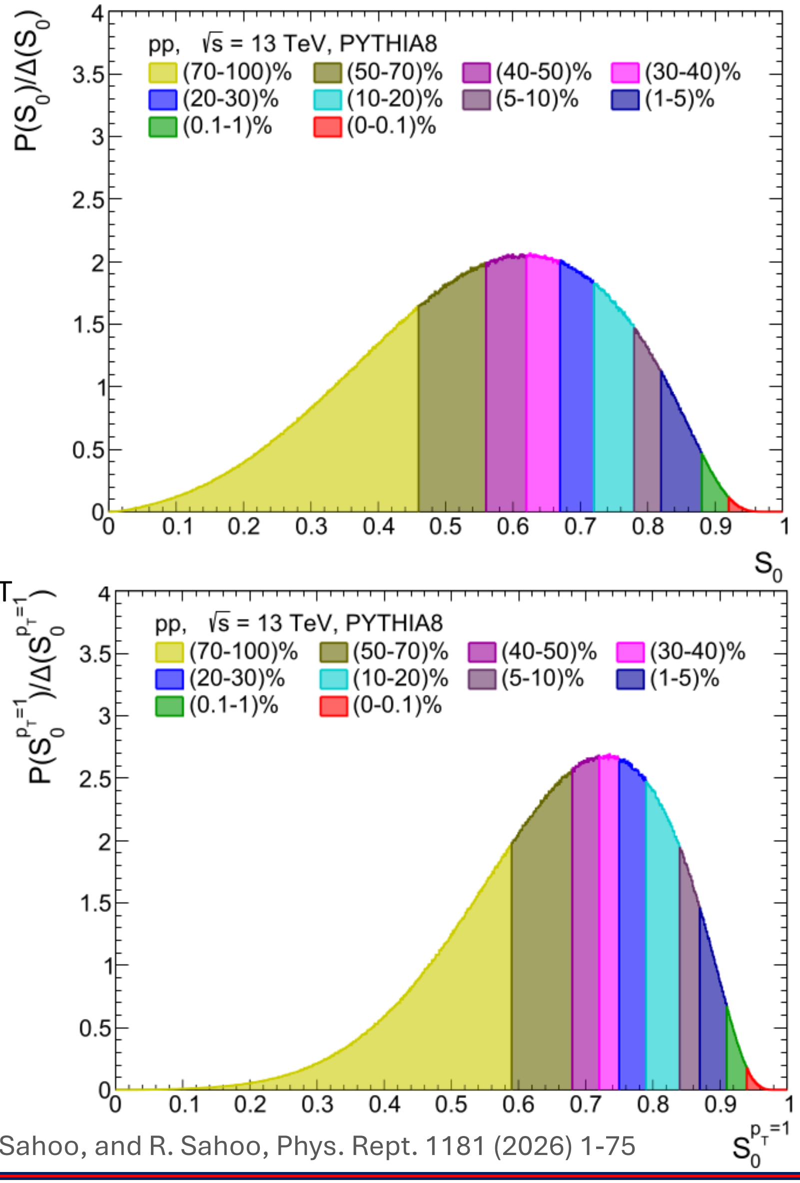
- "Isotropic" limit (Soft-QCD processes)
- "Pencil-like" or "jetty" limit (Hard-QCD processes)



S. Prasad, N. Mallick, S. Tripathy, and R. Sahoo, Phys. Rev. D 107, 074011 (2023)

- An event that has a charged-particle "isotropic" topology might still have a low-p_T jet with a neutral leading particle
- An event that has a charged-particle "jetty" topology might contain a lot of neutral soft production
- Unweighted Spherocity Estimator:

$$S_0^{p_T=1} = \frac{\pi^2}{4} \min_{\hat{n}} \left(\frac{\Sigma_i |\hat{p}_T \times \hat{n}|}{N_{ch}} \right)^2$$



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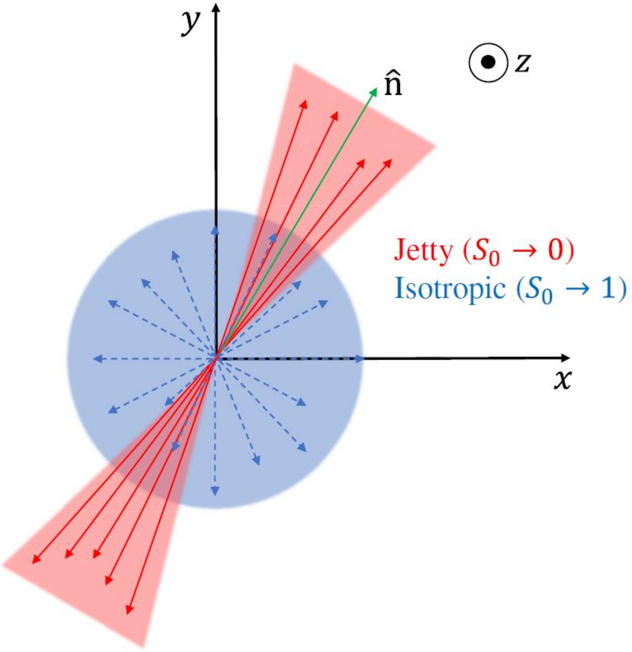

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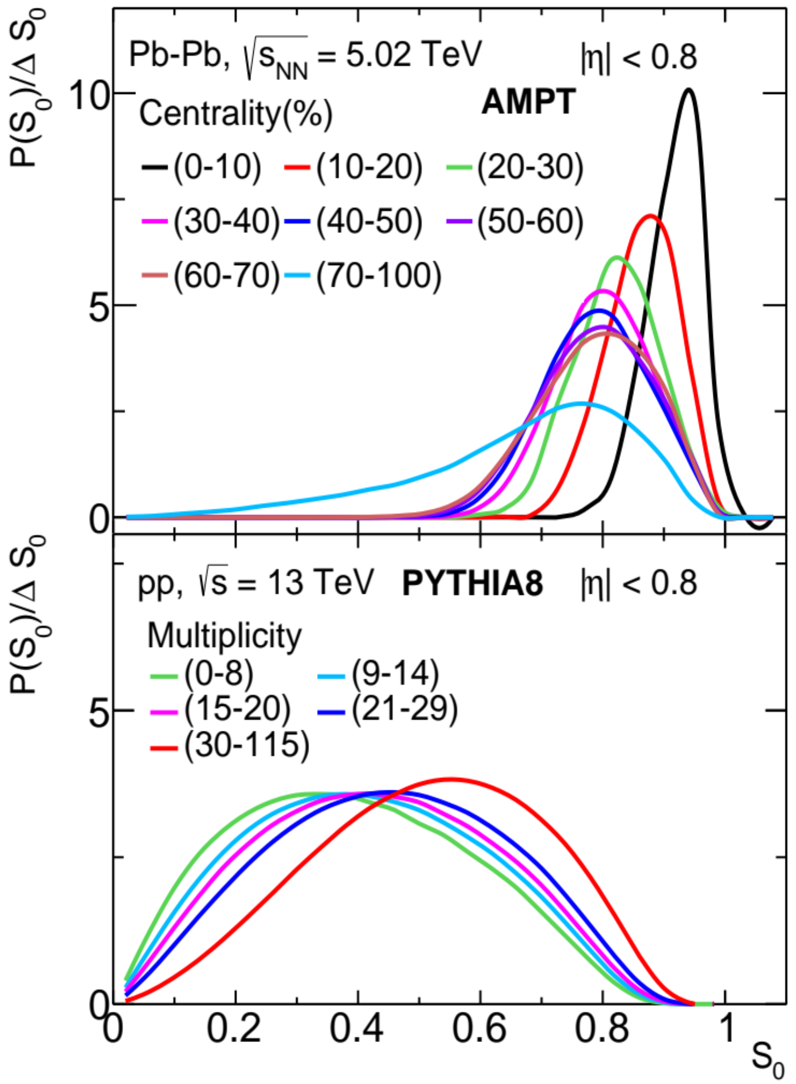

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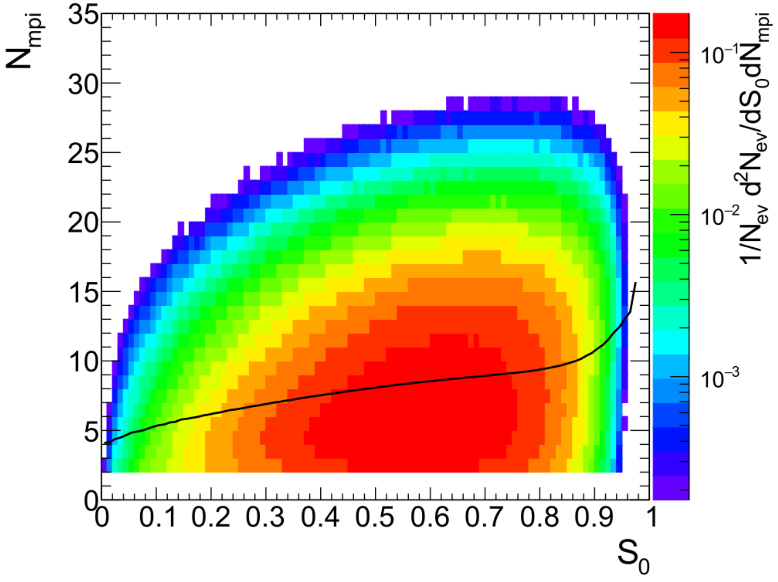
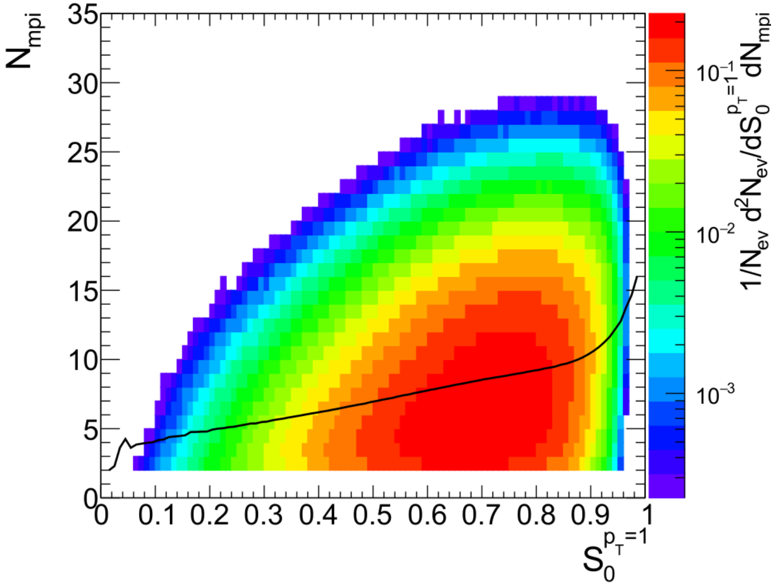
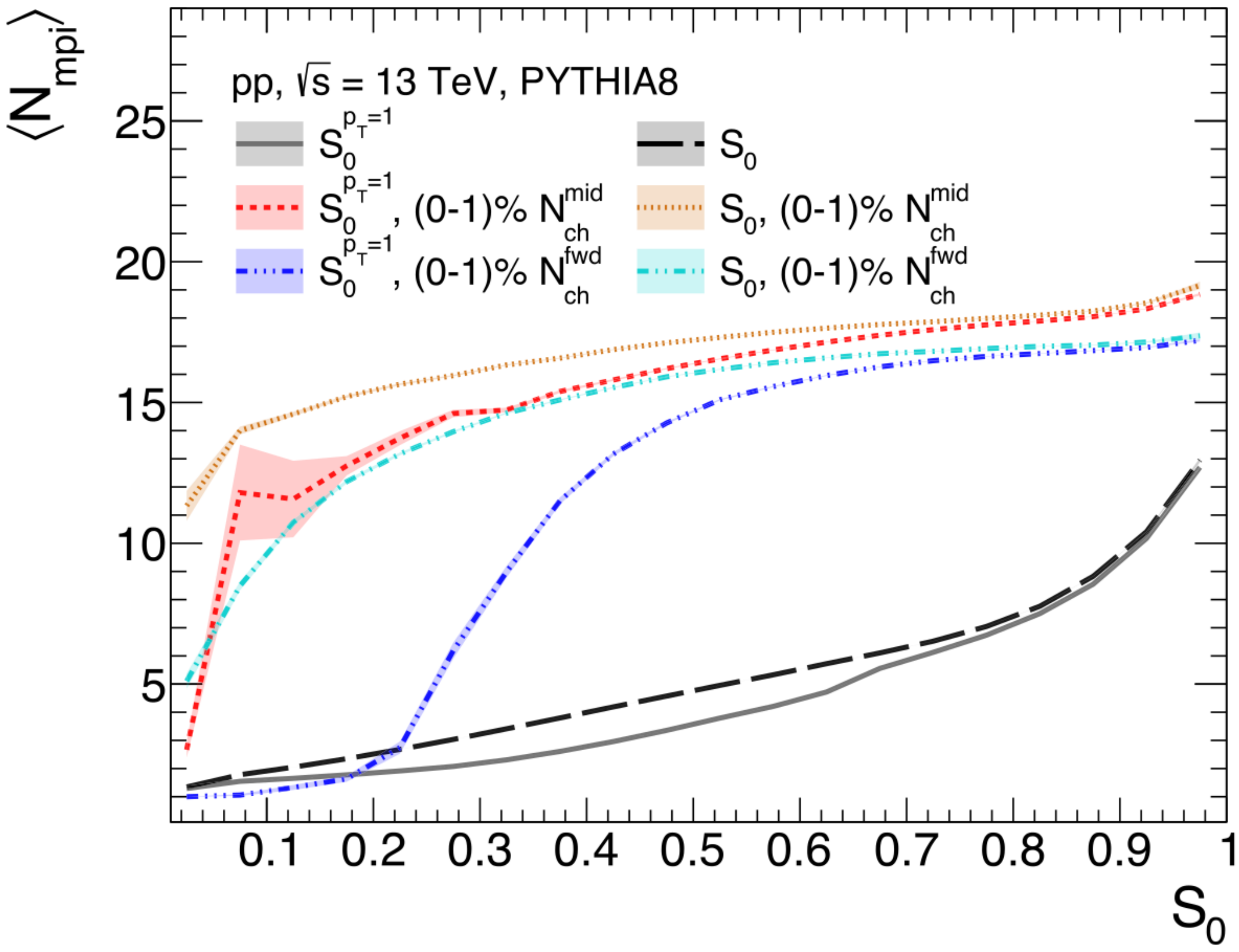
S. Prasad, N. Mallick, D. Behera, R. Sahoo and S. Tripathy, Sci. Rep. 12, 3917 (2022)

Transverse Spherocity vs N_{mpi}







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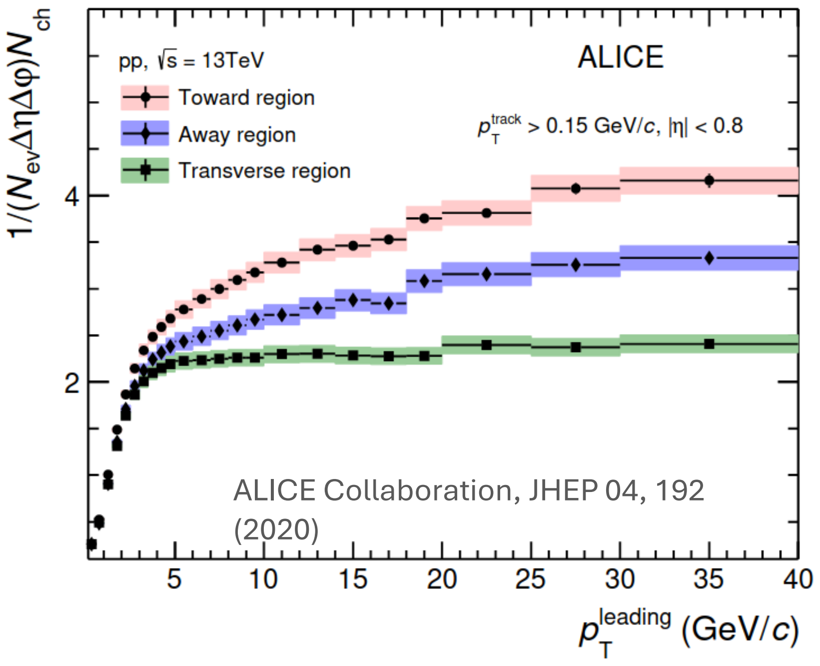
Relative Transverse Activity Classifier (R_T)

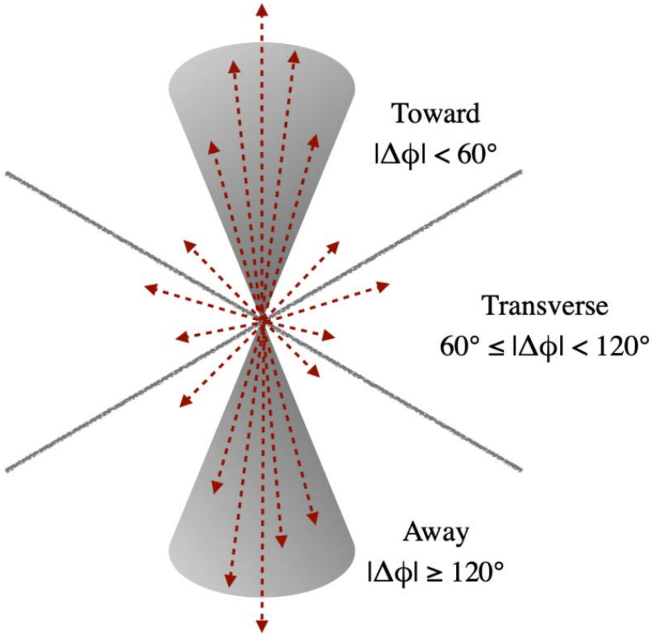
- To quantify the underlying event activity, relative transverse activity classifier (R_{T}) is defined using following expression:

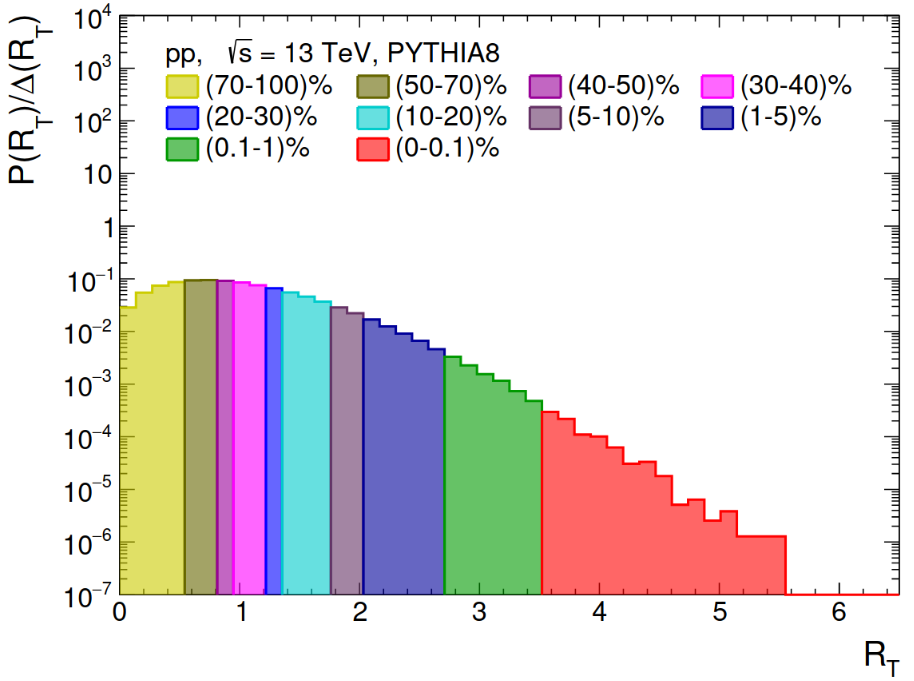
$$R_{\mathrm{T}} = \frac{N_{\mathrm{ch}}^{\mathrm{T}}}{\langle N_{\mathrm{ch}}^{\mathrm{T}} \rangle}$$

- $N_{\mathrm{ch}}^{\mathrm{T}}$ is the multiplicity in the transverse region

- Events with $p_{\mathrm{T}}^{\mathrm{lead}} \geq 5$ GeV/c are considered
- Estimated with charged particles in $|\eta| < 0.8$







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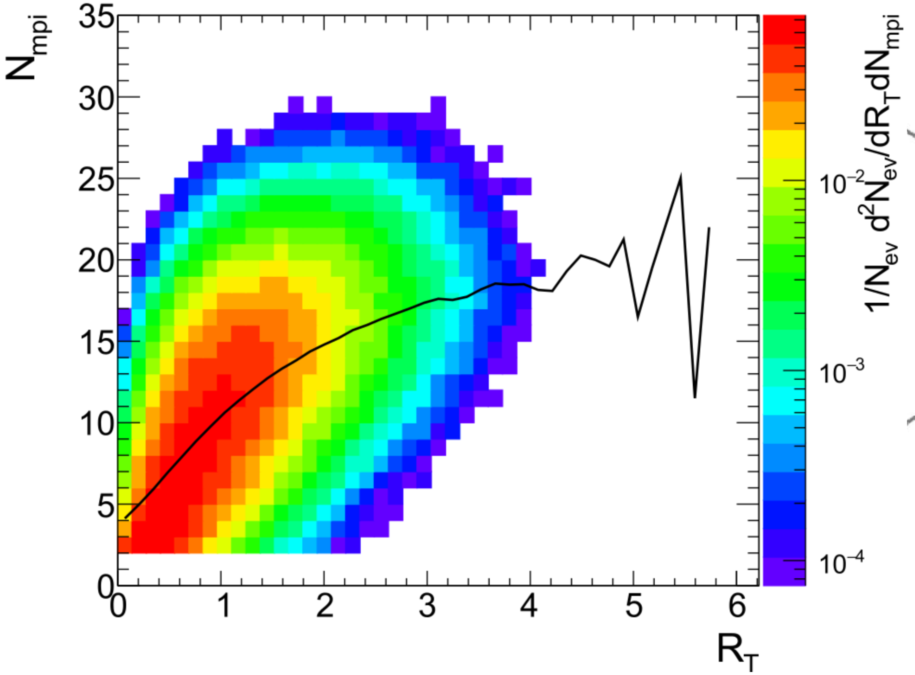

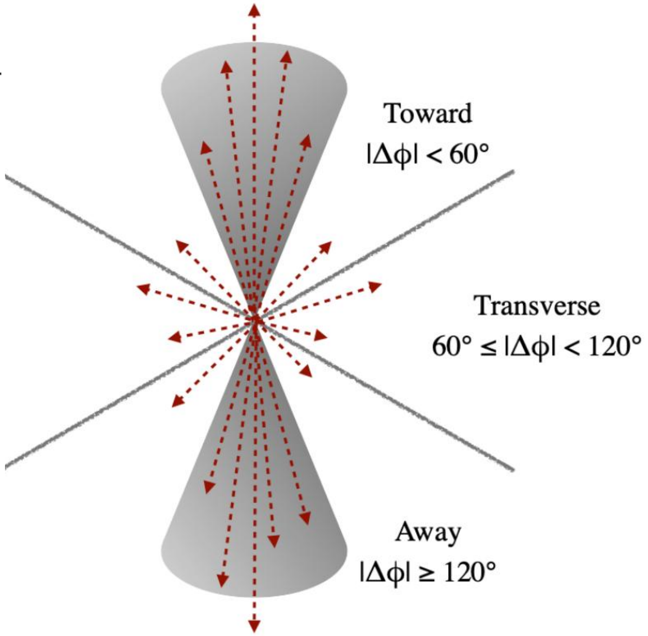

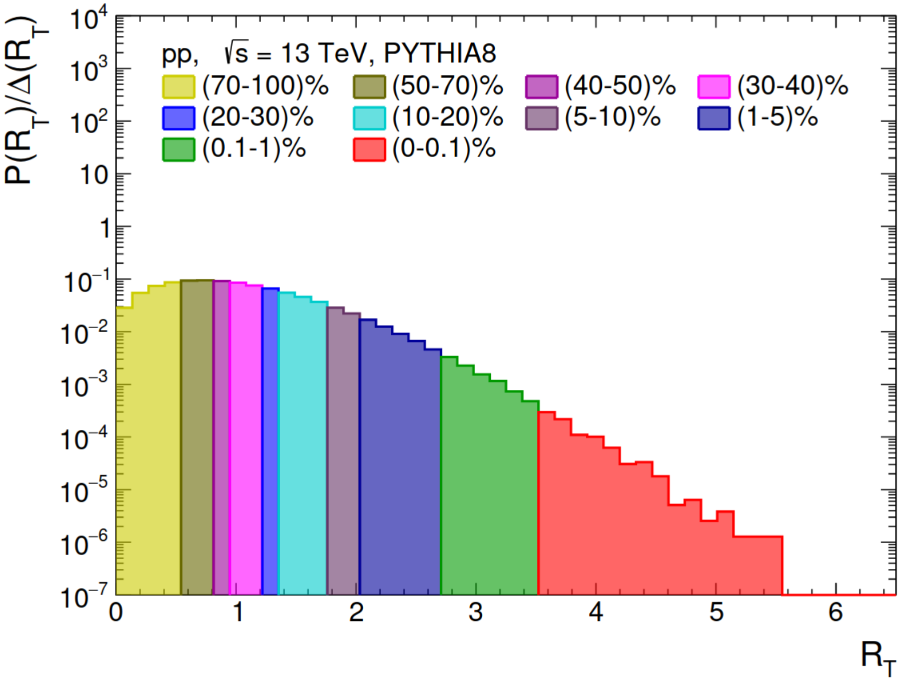

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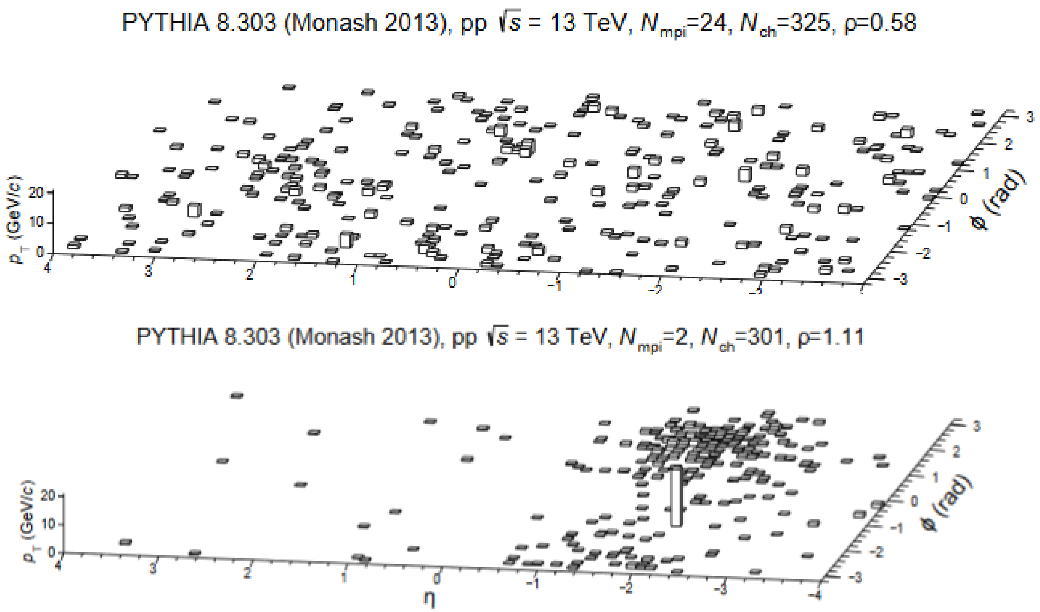
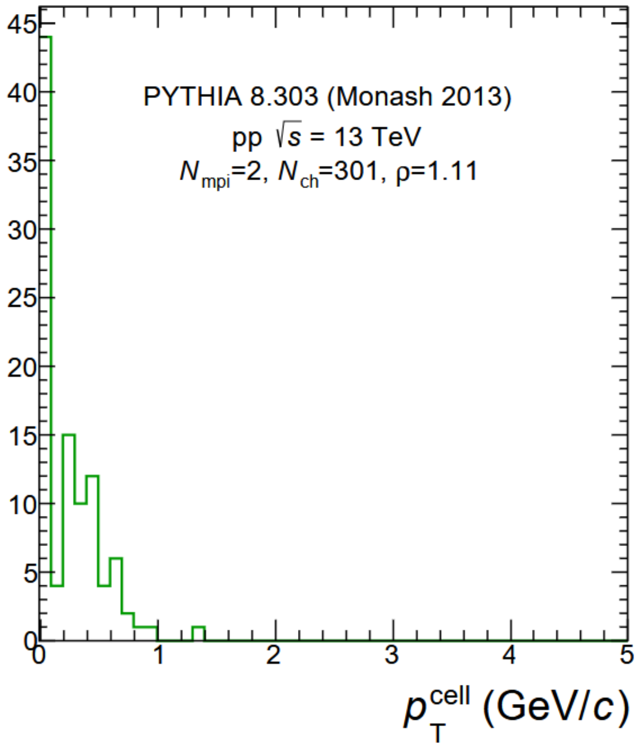
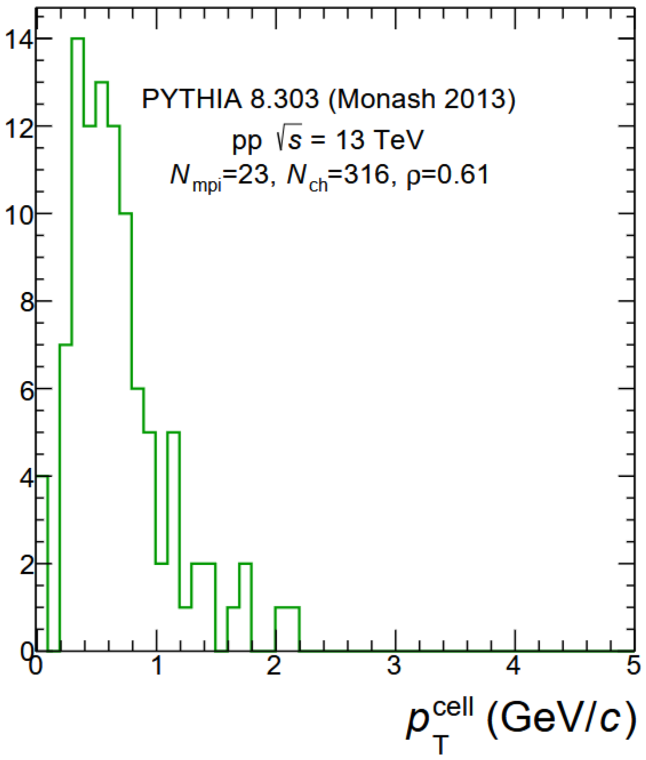
Charged Particle Flattenicity







- All charged particle in $|\eta| < 4$, $p_T > 0.15$ GeV/c region are used (ALICE3)

A. Ortiz, G. Paic, Rev. Mex. Fis. Suppl. 3, 040911 (2022).

- Flattenicity is given as:

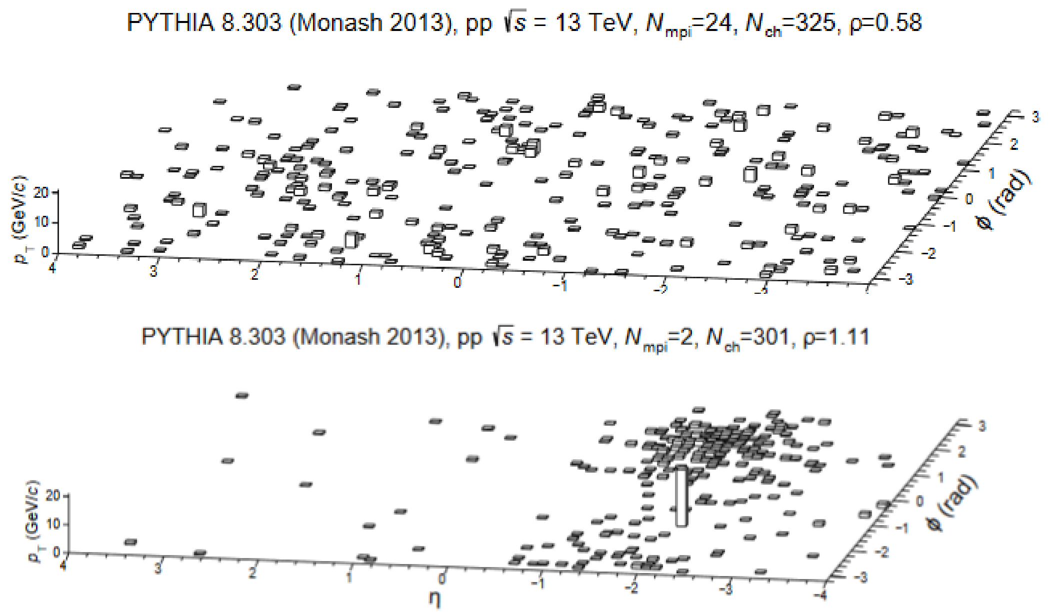
$$\boxed{\rho = \frac{\sigma_{p_{\mathrm{T}}^{\mathrm{cell}}}}{\langle p_{\mathrm{T}}^{\mathrm{cell}} \rangle}}$$

- $\sigma_{p_{\mathrm{T}}^{\mathrm{cell}}}$ → Small for Isotropic Events
- $\sigma_{p_{\mathrm{T}}^{\mathrm{cell}}}$ → Large for jetty events

⇒ $\rho \to 0$: Soft Events

⇒ $\rho \to 1$: Hard Events

Charged Particle Flattenicity



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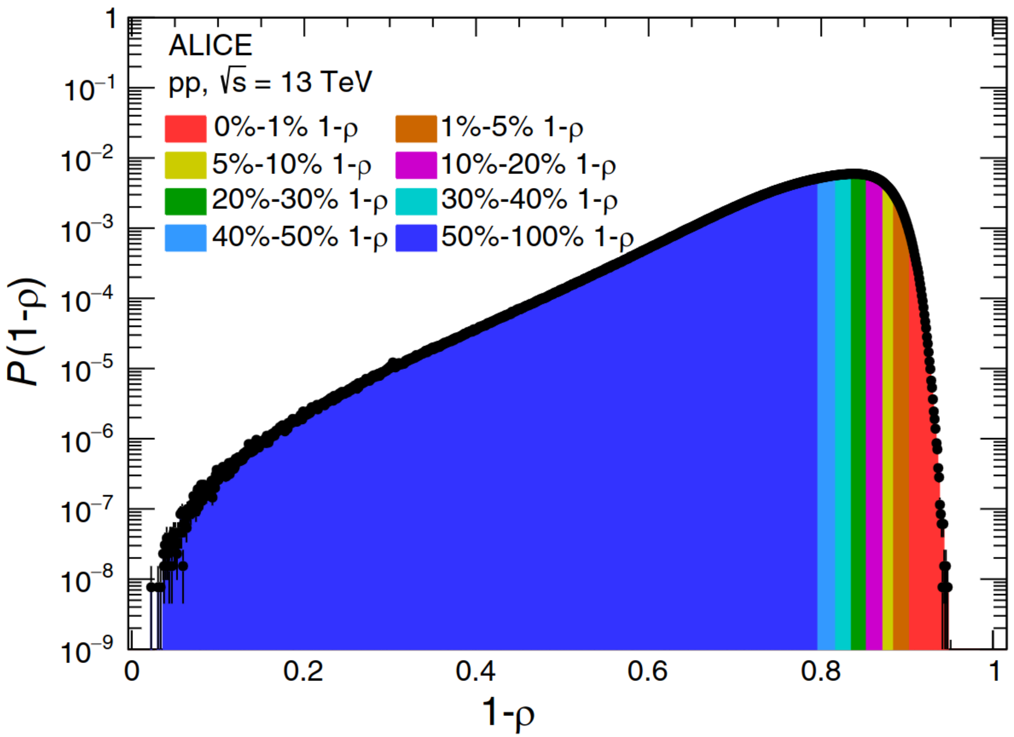
A. Ortiz et. al., Phys. Rev. D 107, 076012 (2023).

ALICE, Phys. Rev. D.111, 012010 (2025)

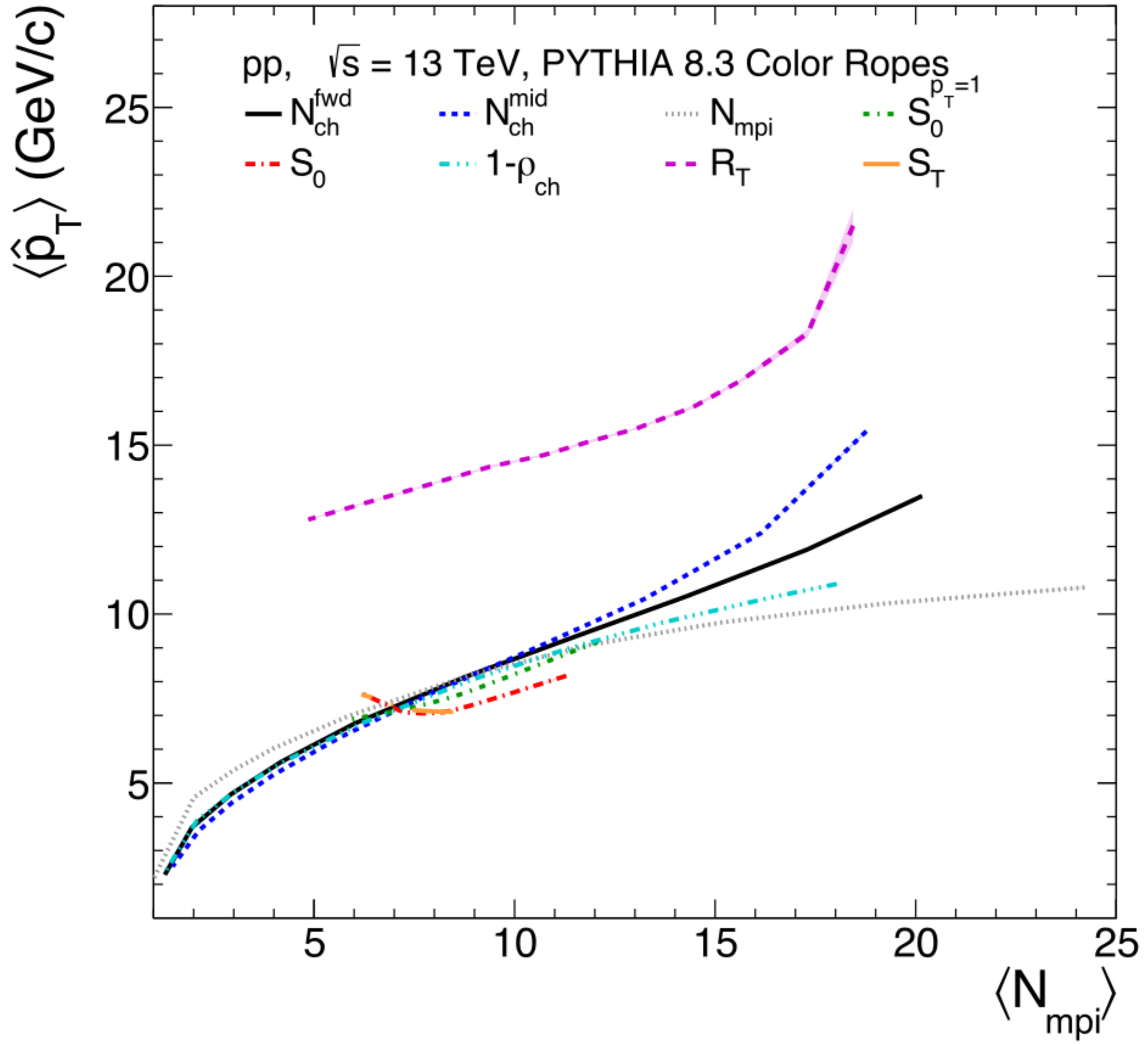
$$\rho_{ch} = \frac{\sqrt{\sum_i \left(N_{ch}^{cell,i} - \langle N_{ch}^{cell} \rangle\right)^2 / N_{cell}^2}}{\langle N_{ch}^{cell} \rangle}$$

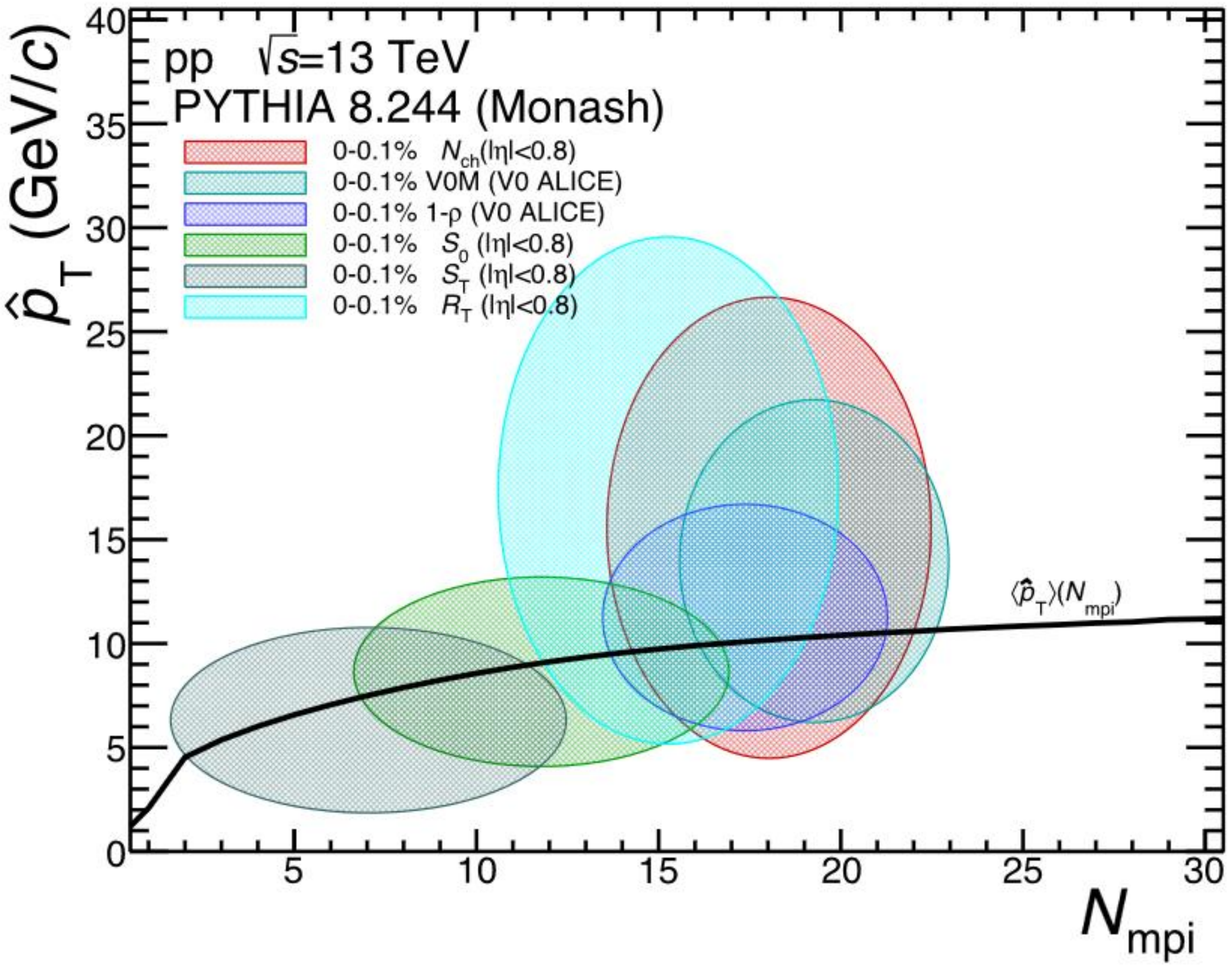
$\rho_{ch} \rightarrow 0$: Soft Events
$\rho_{ch} \rightarrow 1$: Hard Events

Measured with ALICE V0
(−3.7 < η < −1.7 and 2.8 < η < 5.1)



Correlation with N_{mpi}





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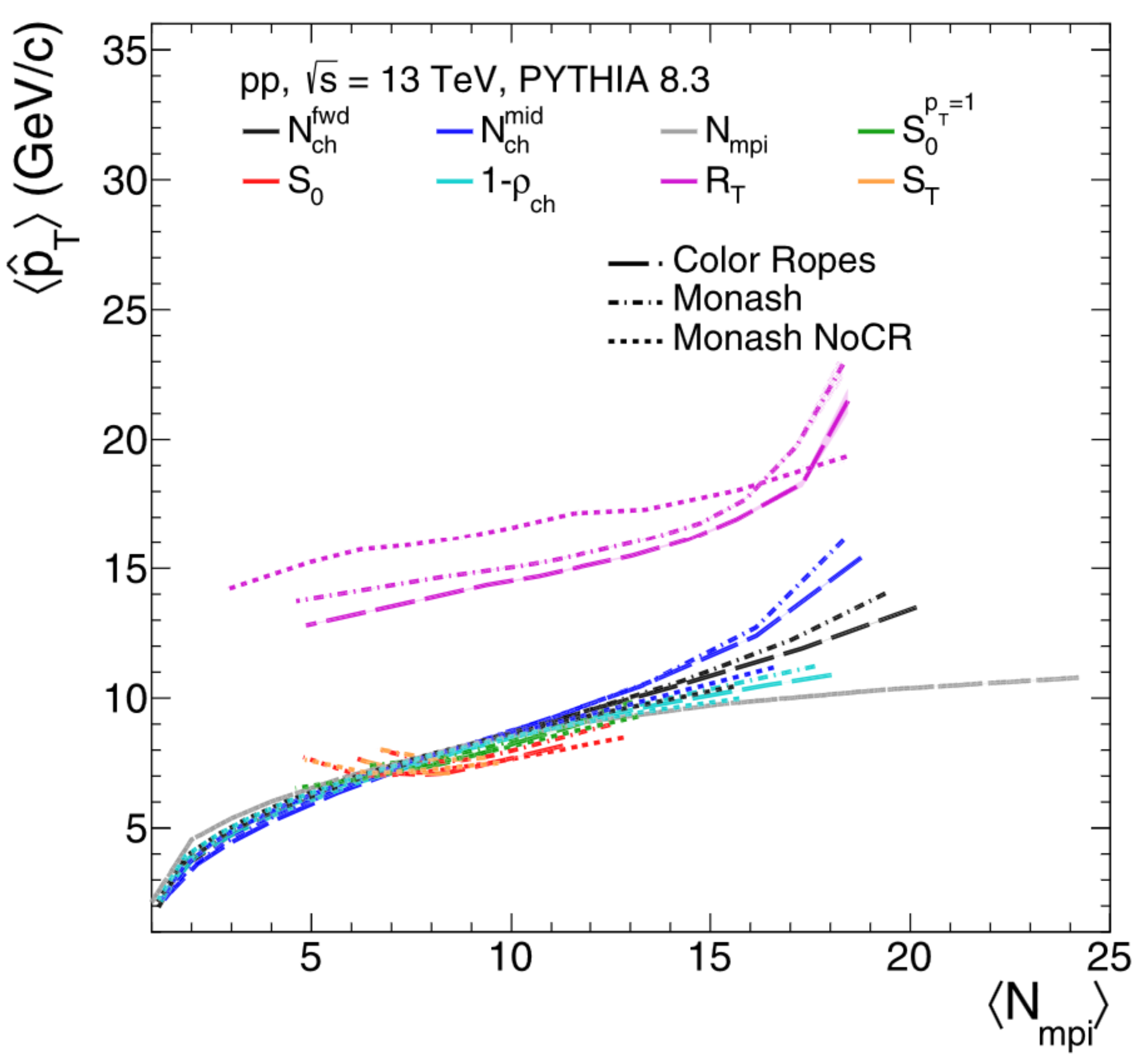
J. E. M. Méndez and A. Ortiz, J. Phys. G: Nucl. Part. Phys. 52 (2025) 095001

Correlation with N_{mpi}



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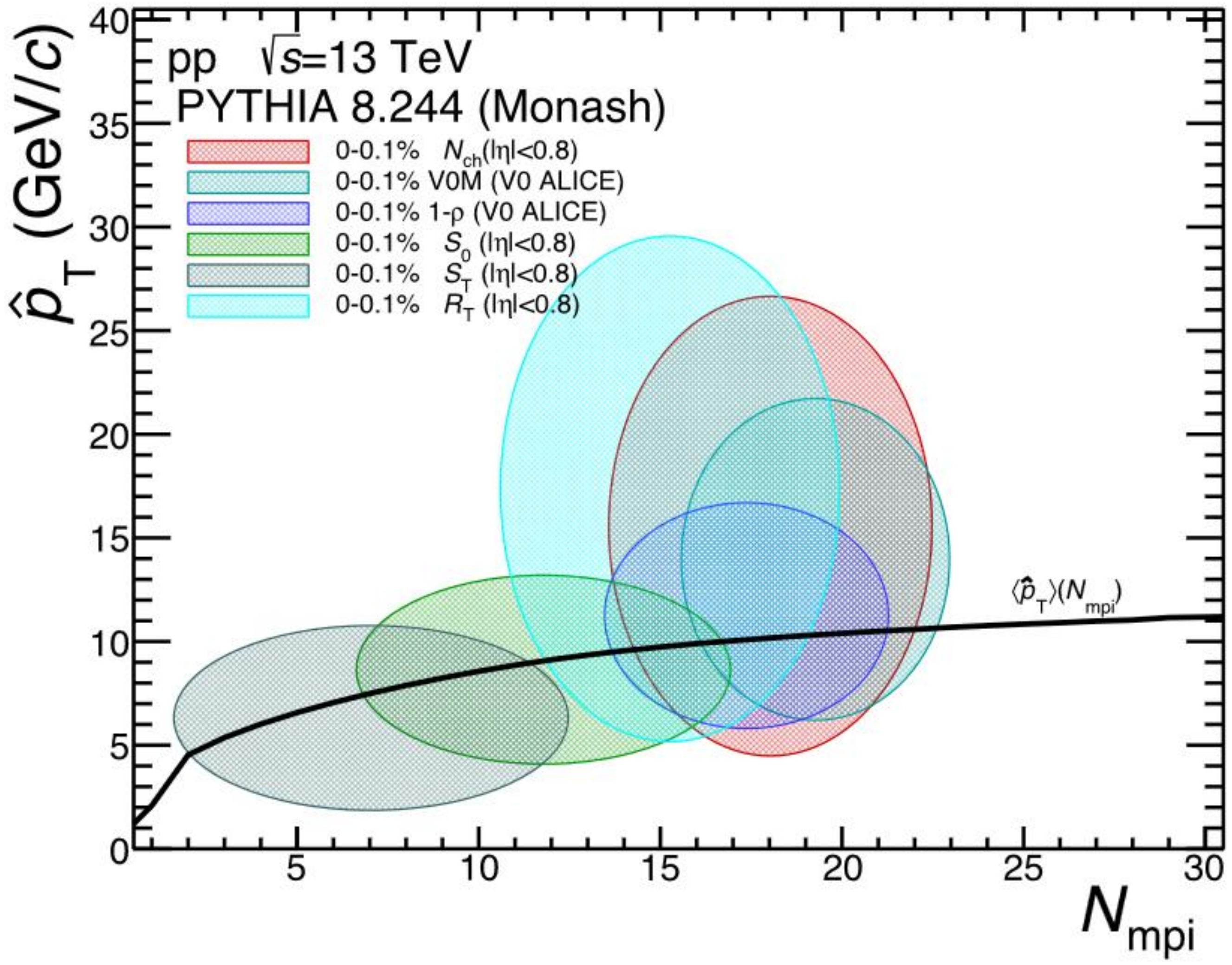

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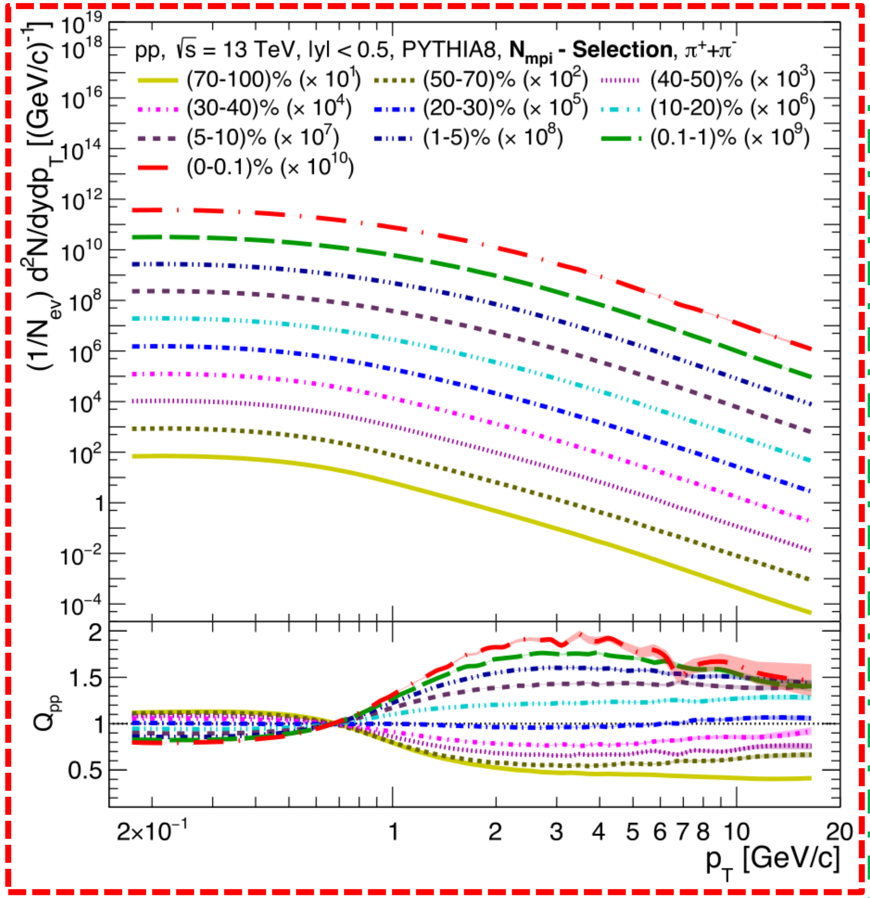
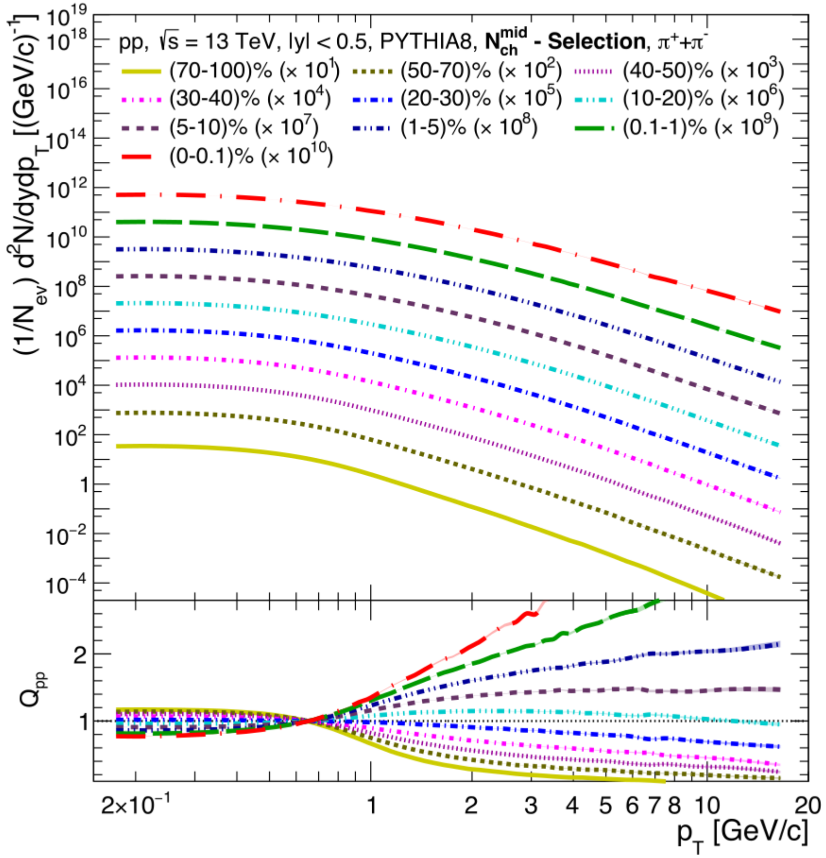
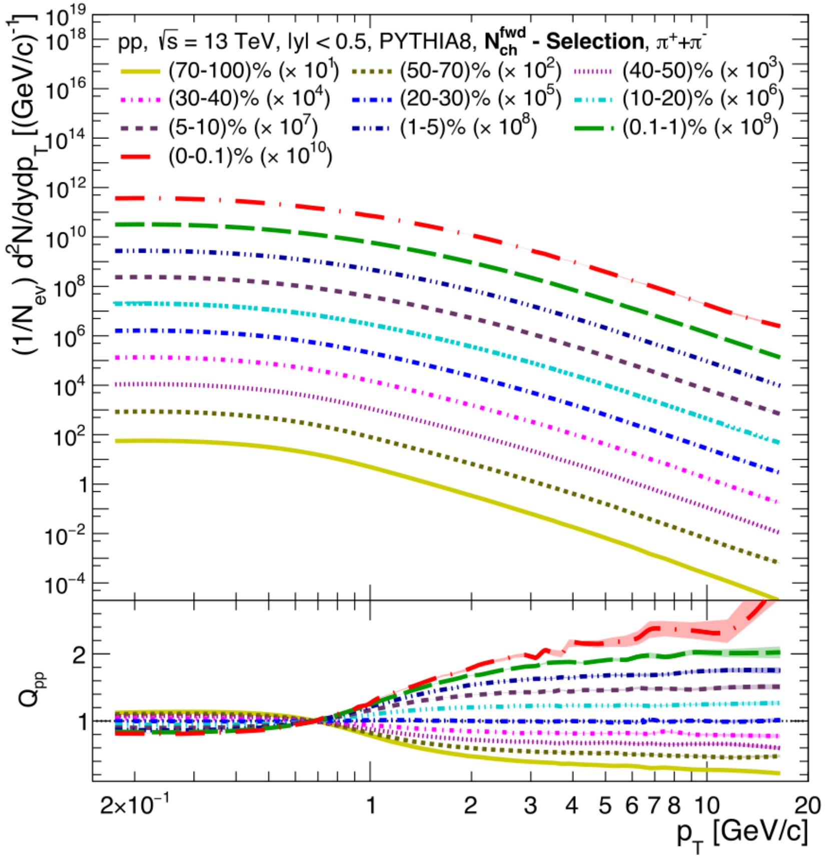
Looking at the Biases

p_T spectra and Q_{pp}



- A good event classifier should behave close to N_{mpi}





$$Q_{\mathrm{pp}} = \frac{d^2 N_{\mathrm{ch}}^{\mathrm{ES}} / \langle N_{\mathrm{ch}}^{\mathrm{ES}} \rangle d\eta dp_{\mathrm{T}}}{d^2 N_{\mathrm{ch}}^{\mathrm{ES\text{-}int}} / \langle N_{\mathrm{ch}}^{\mathrm{ES\text{-}int}} \rangle d\eta dp_{\mathrm{T}}}$$

- $N_{\mathrm{ch}}^{\mathrm{mid}}$ based selection is highly biased
- $N_{\mathrm{ch}}^{\mathrm{fwd}}$ based selection relatively less biased

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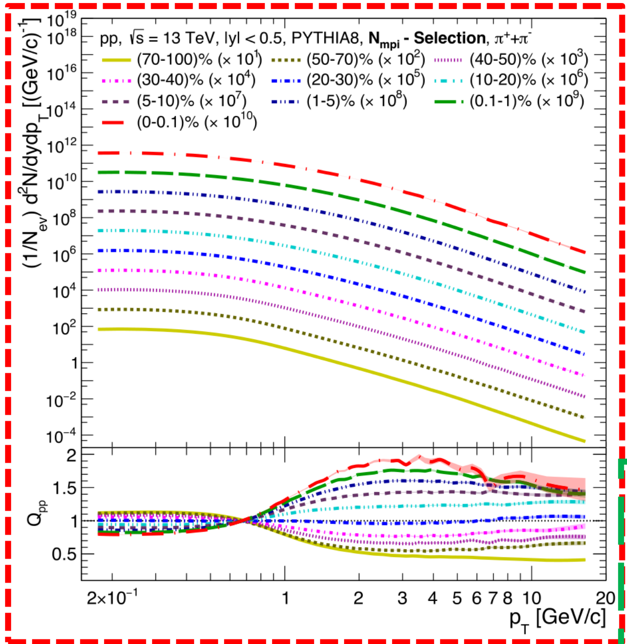
p_T spectra and Q_{pp}



- S_T, S_0 and $S_0^{p_T=1}$ behave similar to each other but completely different from N_{mpi}
- Possible artifact of Jet bias —> Visible in (70-100)%
- Classifiers and charged particle spectra are measured in similar pseudorapidity region

$$Q_{pp} = \frac{d^2N_{ch}^{ES}/\langle N_{ch}^{ES}\rangle d\eta dp_T}{d^2N_{ch}^{ES\text{-}int}/\langle N_{ch}^{ES\text{-}int}\rangle d\eta dp_T}$$

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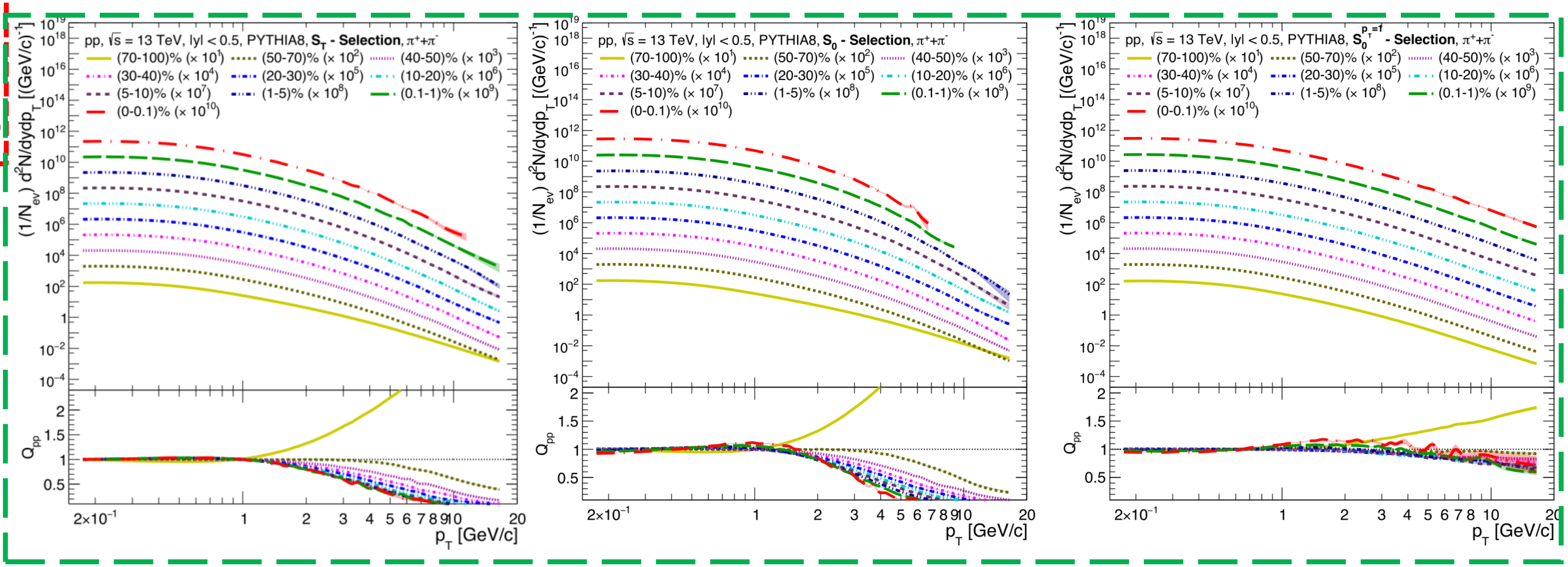

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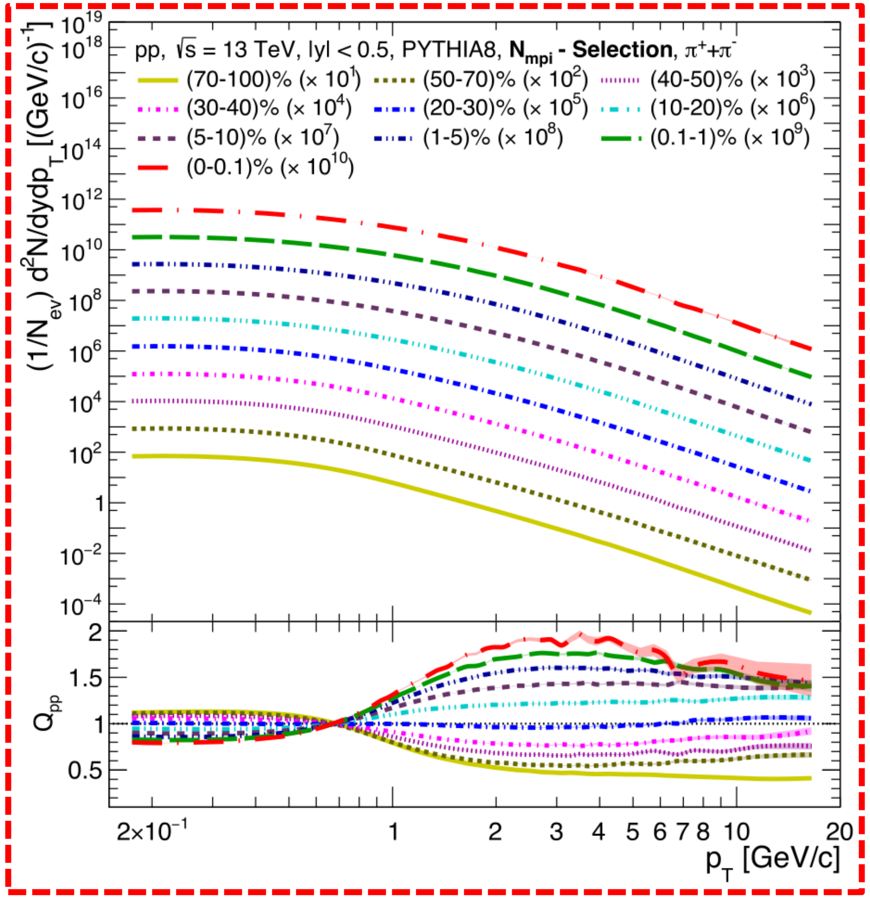

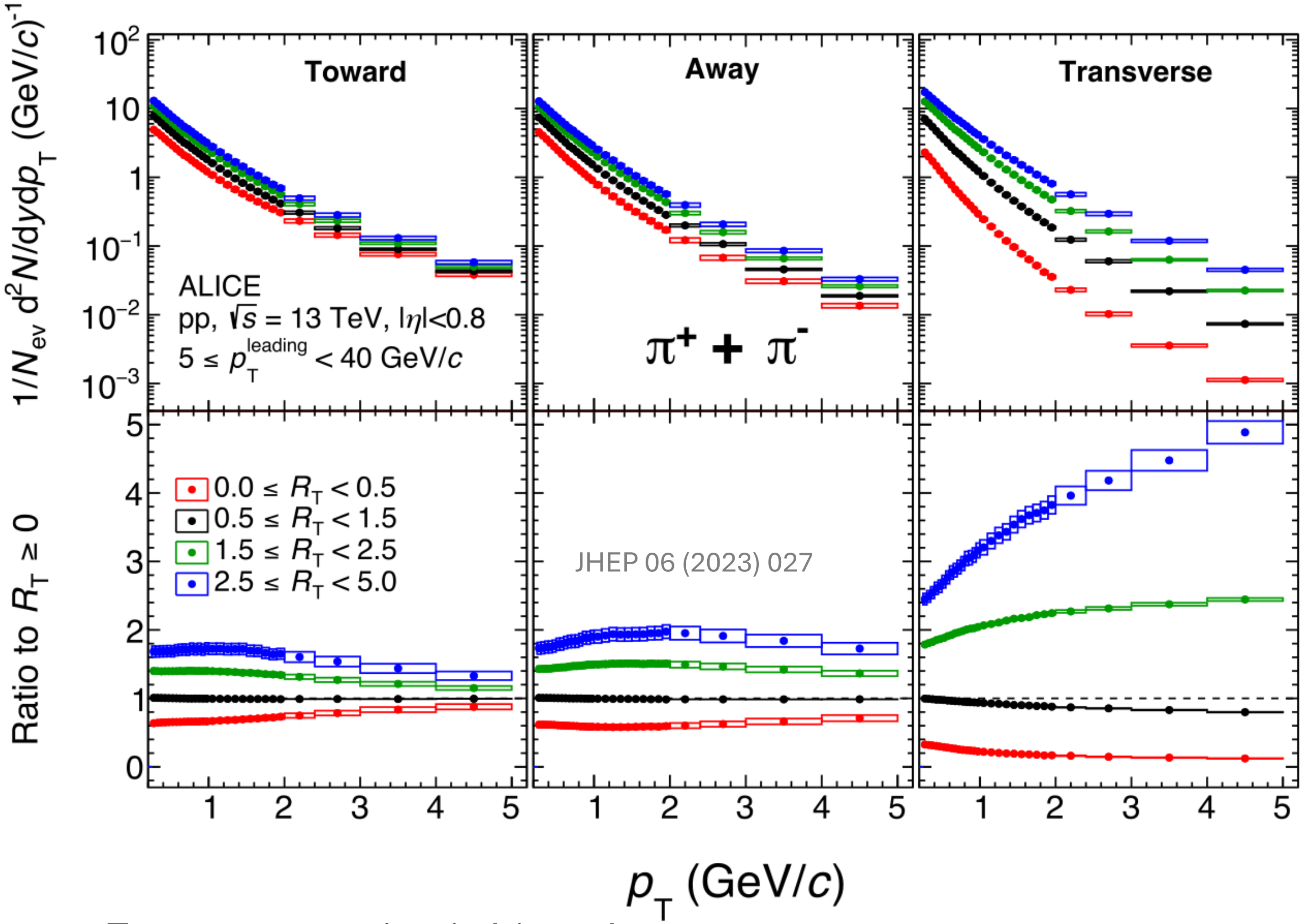

$$Q_{pp} = \frac{d^2N_{ch}^{ES}/\langle N_{ch}^{ES}\rangle d\eta dp_T}{d^2N_{ch}^{ES\text{-}int}/\langle N_{ch}^{ES\text{-}int}\rangle d\eta dp_T}$$

- Transverse region is biased
- Toward and Away region behave similar to N_{mpi}

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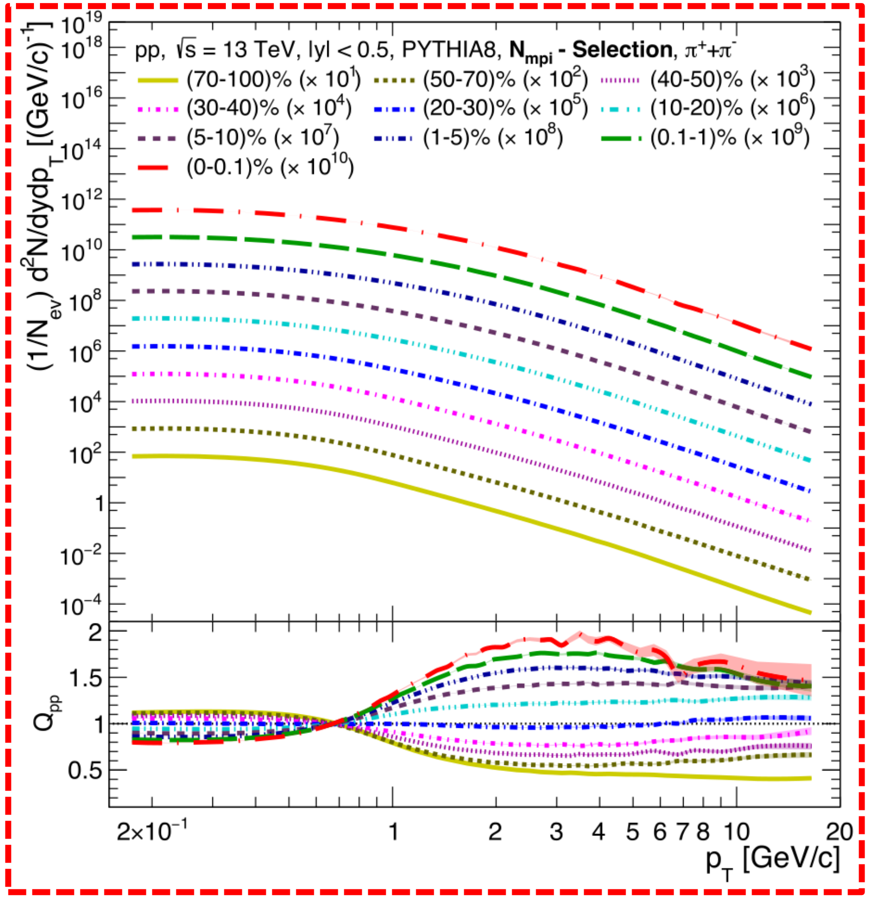
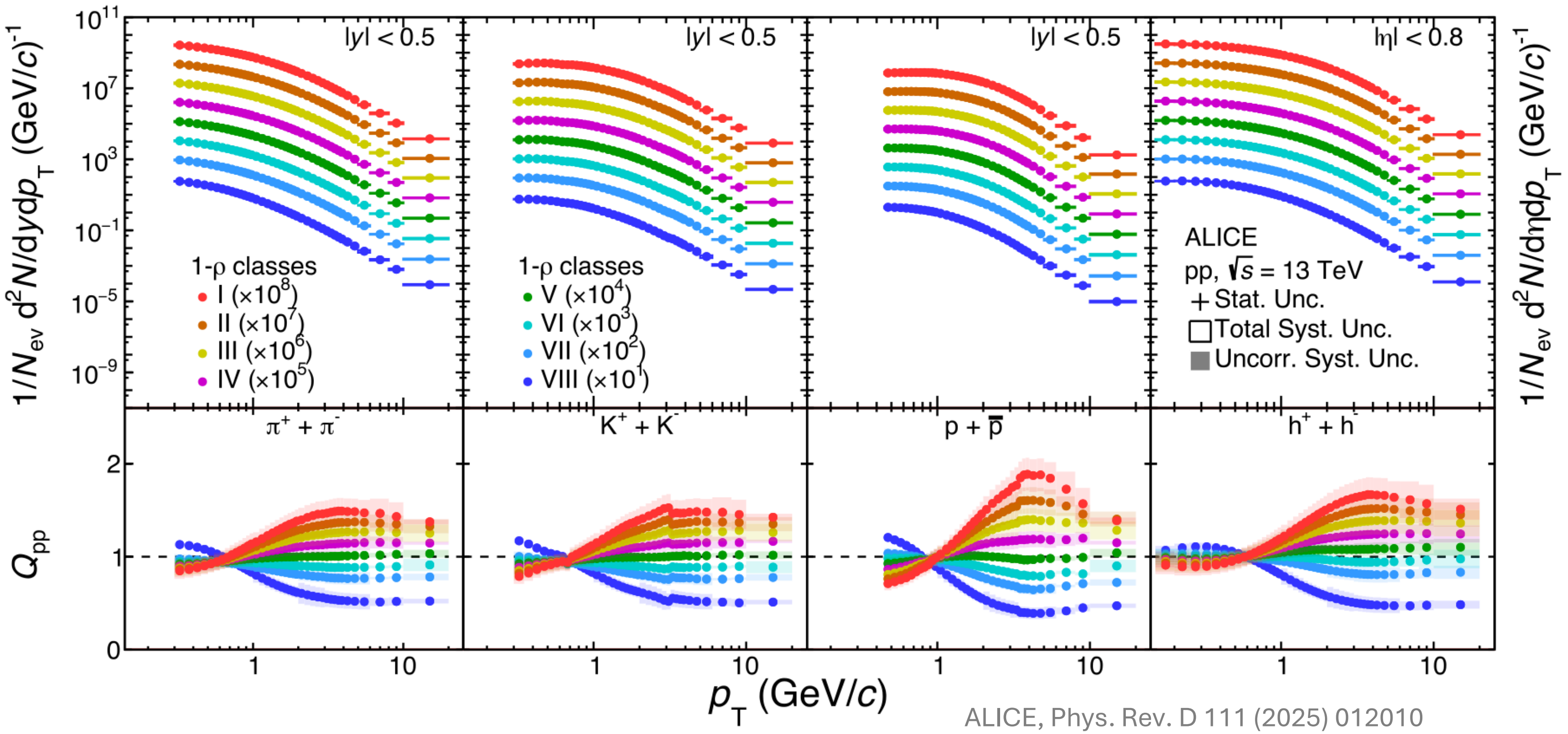
p_T spectra and Q_{pp}





ALICE, Phys. Rev. D 111 (2025) 012010

- Event selection with $1-\rho_{ch}$ behaves like N_{mpi}
- Radial flow like effects are stronger for protons than pions

$$Q_{\mathrm{pp}} = \frac{d^2N_{\mathrm{ch}}^{\mathrm{ES}}/\langle N_{\mathrm{ch}}^{\mathrm{ES}}\rangle d\eta dp_{\mathrm{T}}}{d^2N_{\mathrm{ch}}^{\mathrm{ES\text{-}int}}/\langle N_{\mathrm{ch}}^{\mathrm{ES\text{-}int}}\rangle d\eta dp_{\mathrm{T}}}$$

Study particle species dependence of these effects

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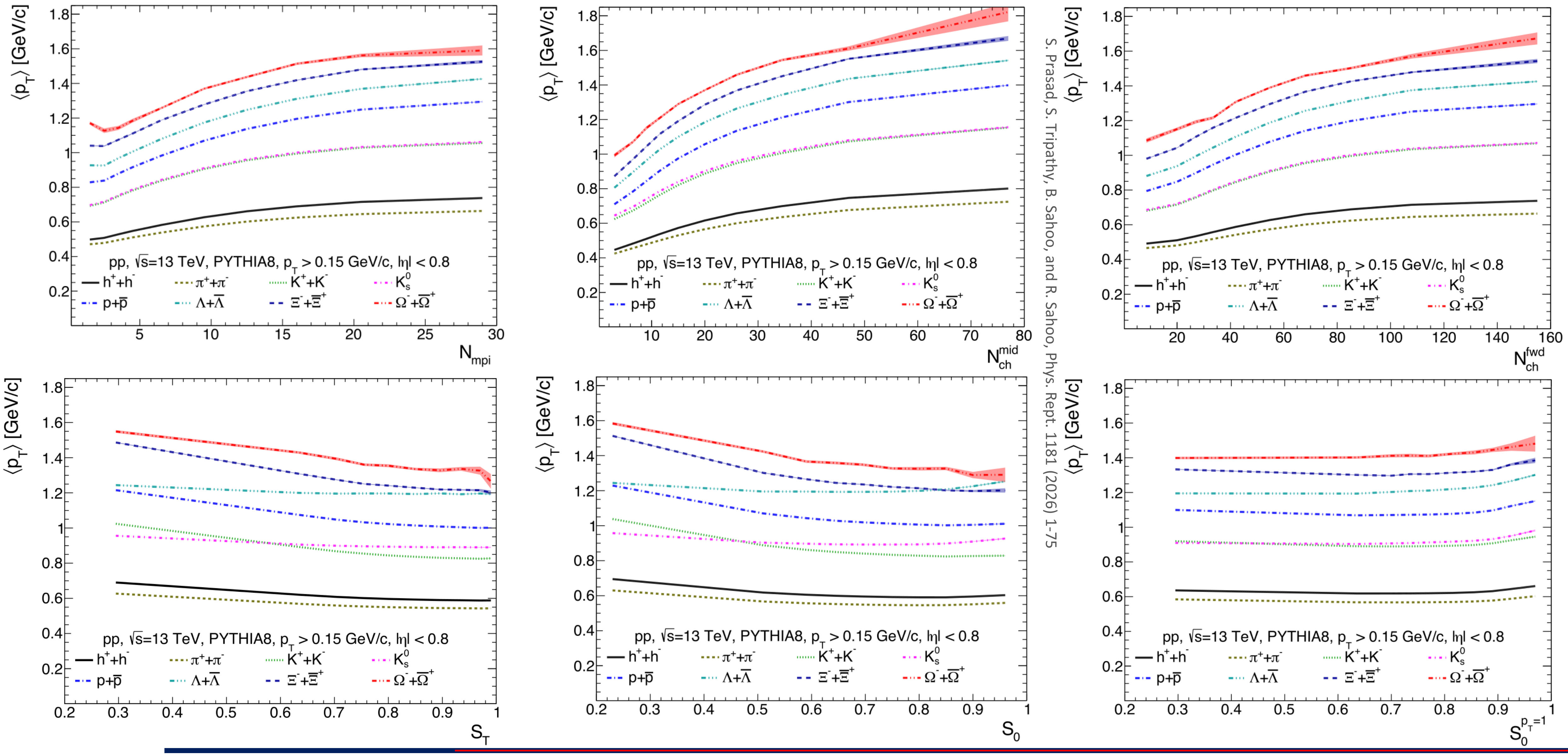
Average Transverse Momentum in each classifier



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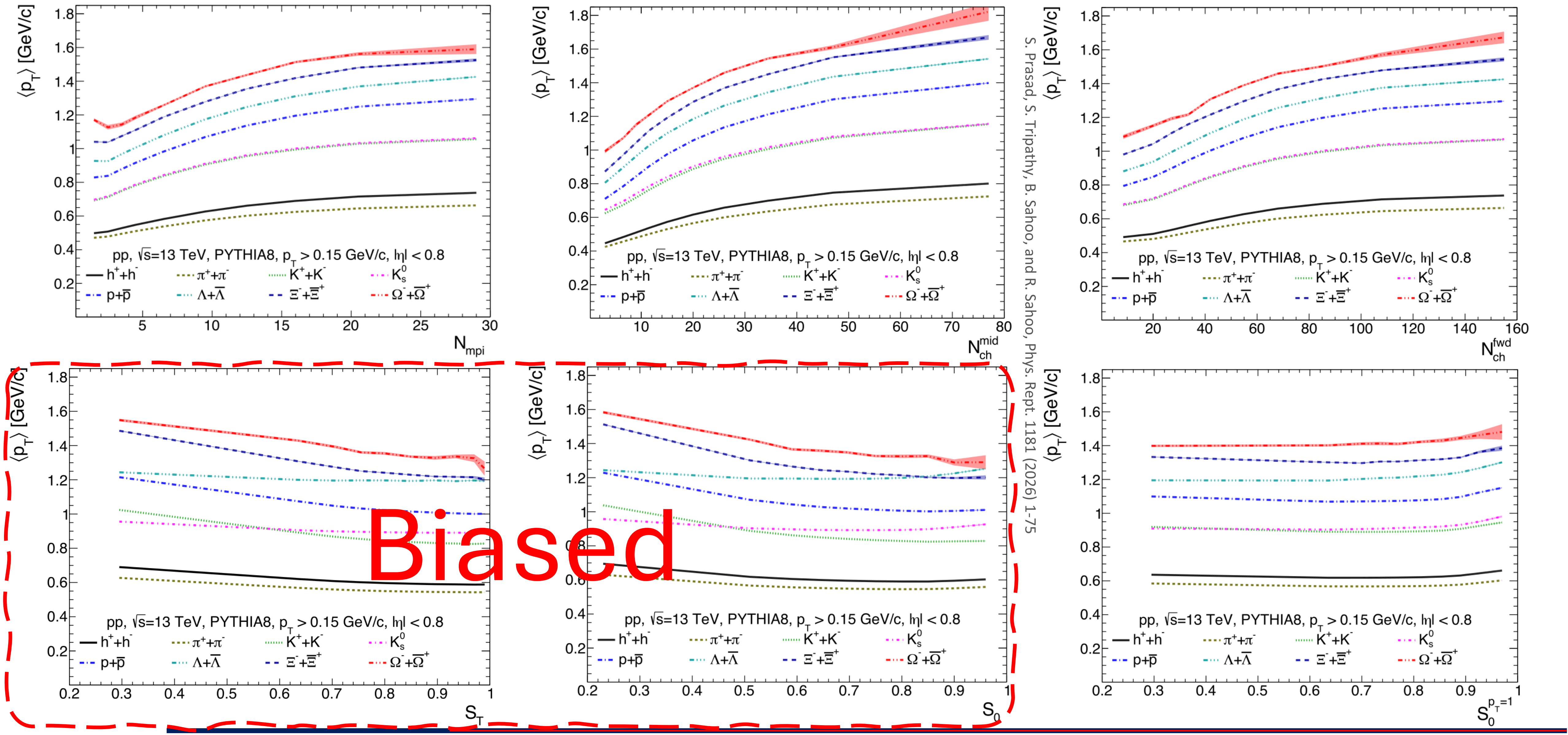
Average Transverse Momentum in each classifier



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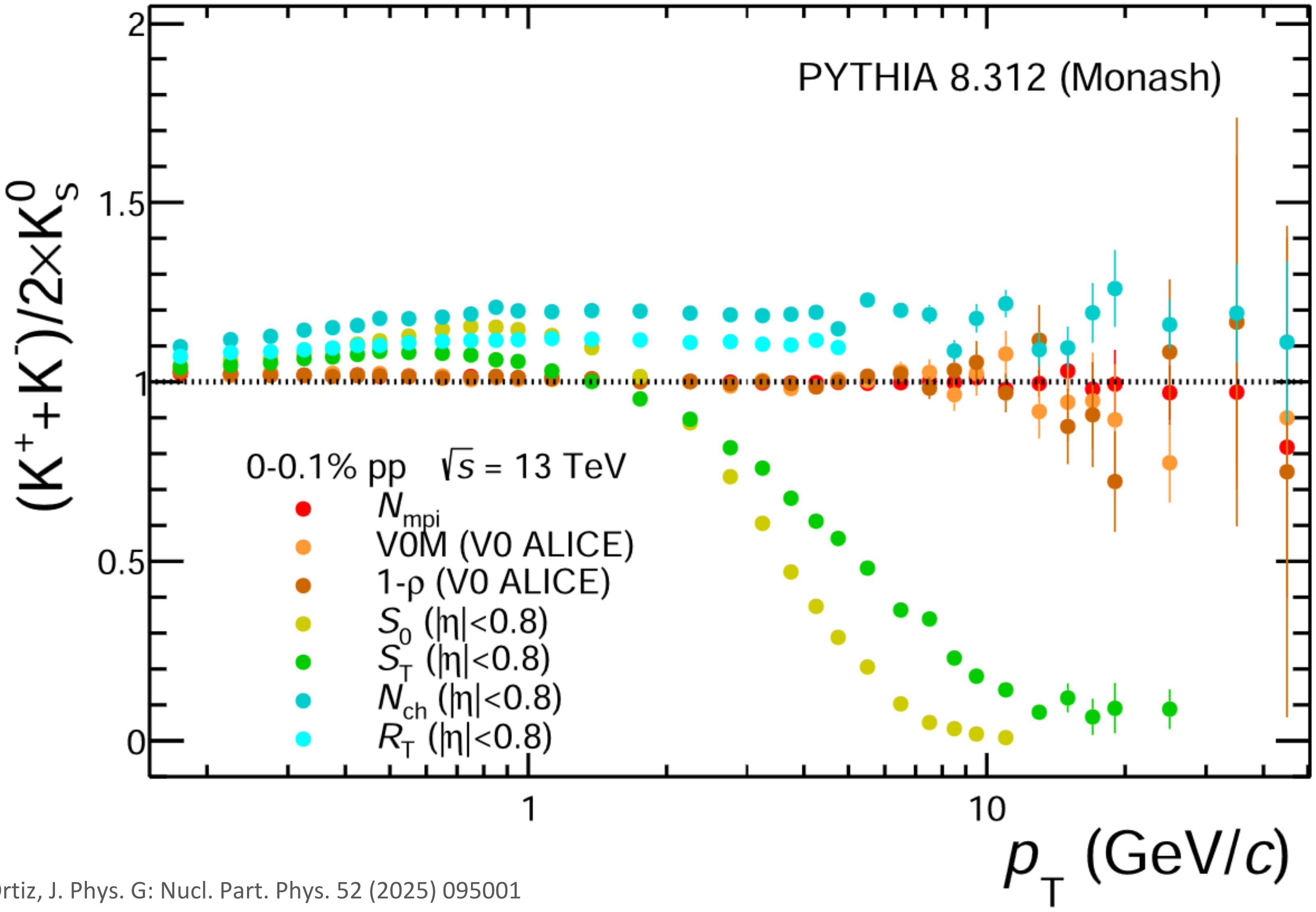
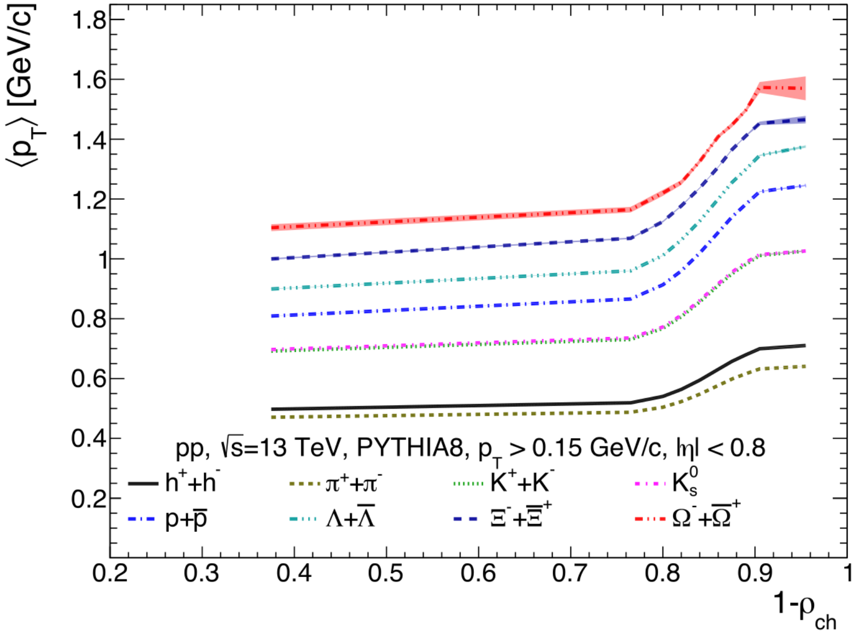
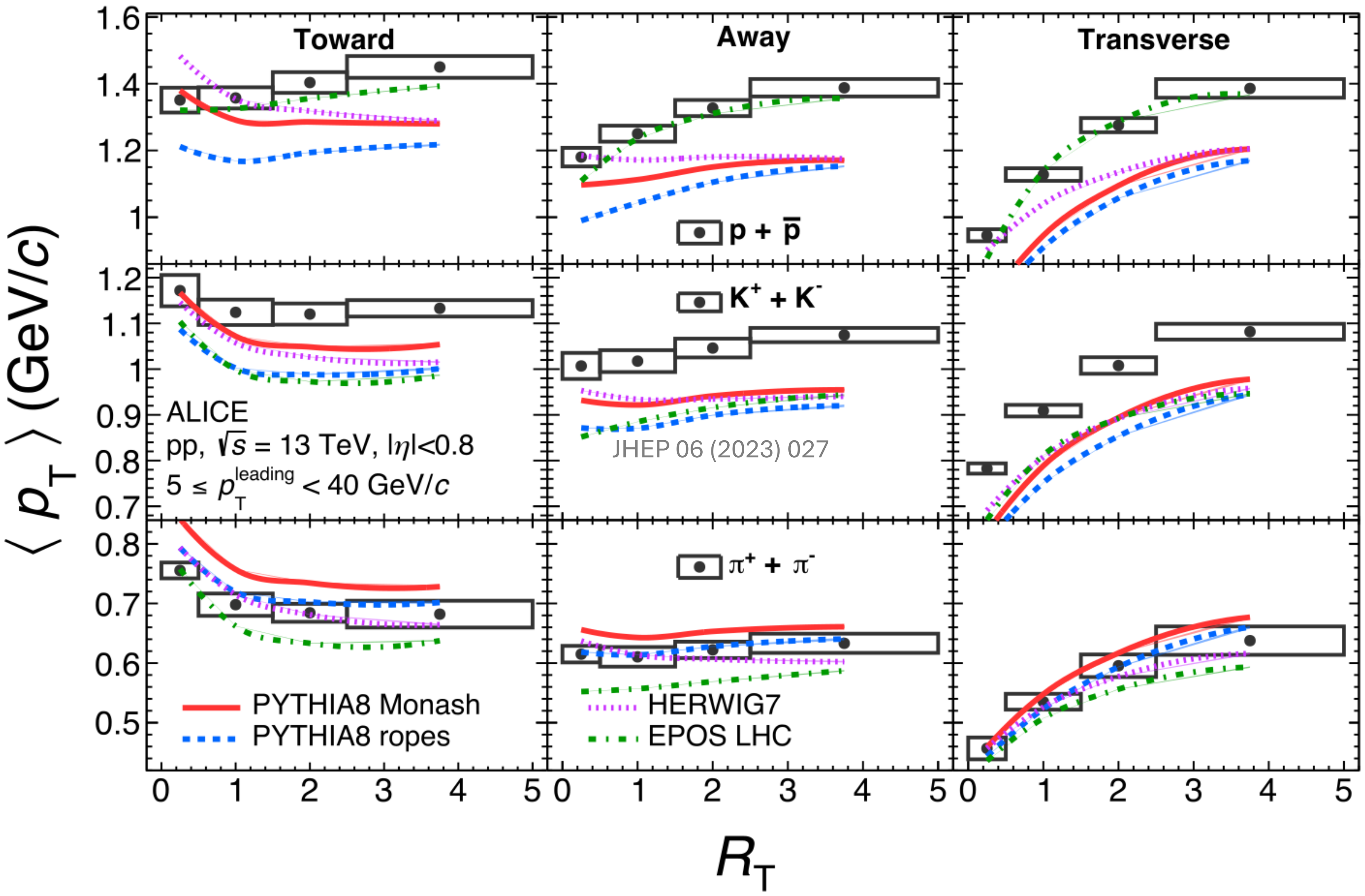
Neutral Bias



J. E. M. Méndez and A. Ortiz, J. Phys. G: Nucl. Part. Phys. 52 (2025) 095001

Average Transverse Momentum in each classifier





- Flattenicity reproduces the particle mass dependence of avg. p_{T}
- Toward region of pions and kaons are contributed from jet fragmentations
- Away region : Competing effects of N_{mpi} and Jet Fragmentations

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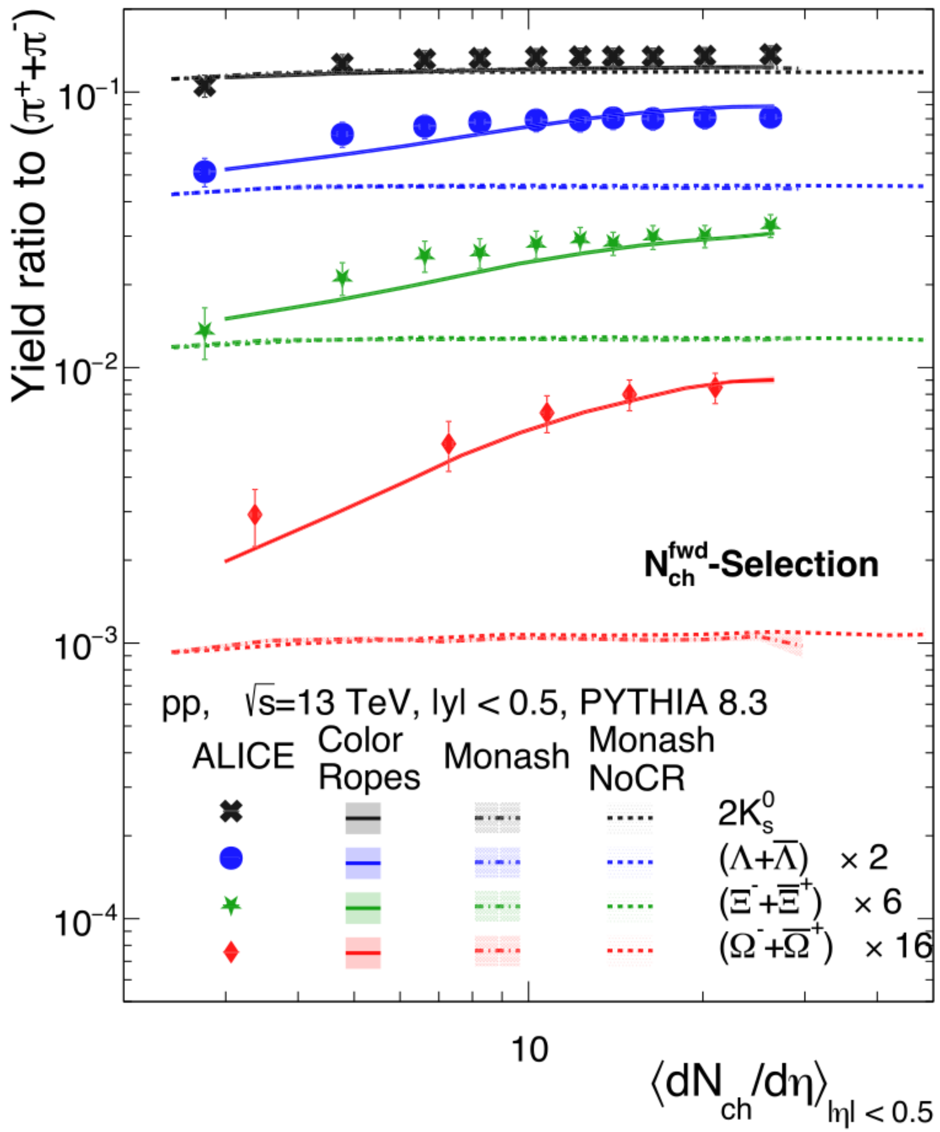
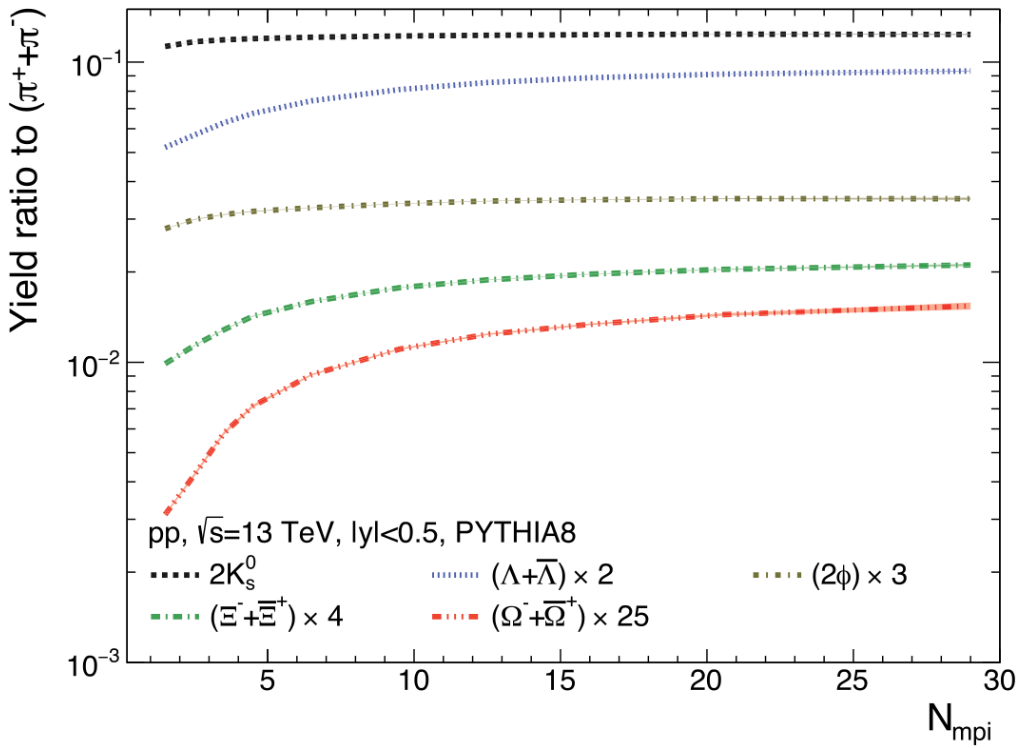
Particle yield ratios to charged pions





- Stronger increasing slope for particles with higher strangeness content
- PYTHIA Color Ropes explains the data
- MPI based features are reflected in $N_{\mathrm{ch}}^{\mathrm{fwd}}$ based selections

- Remember: Enhancement effect saturates at large N_{mpi}

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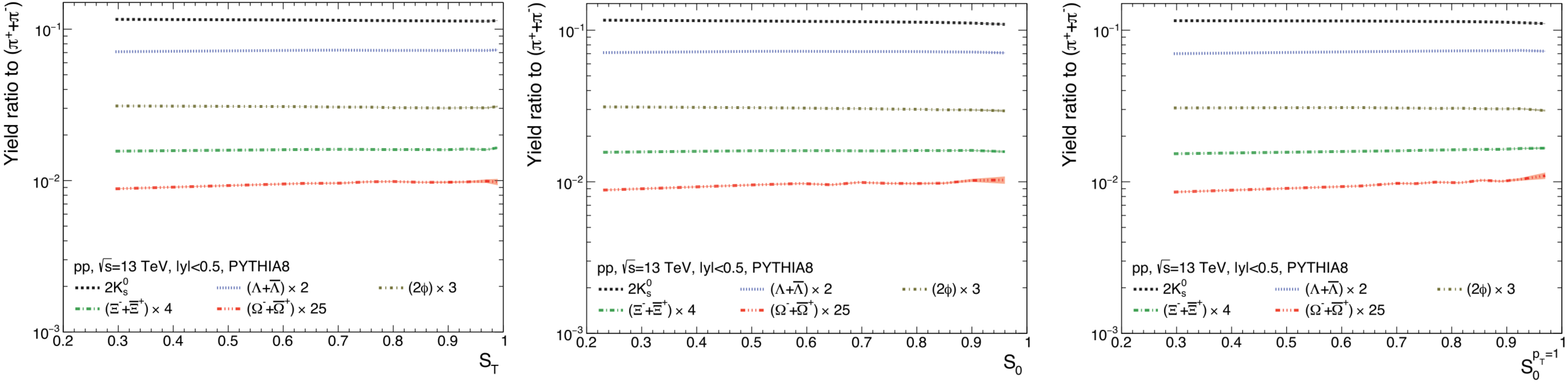
Particle yield ratios to charged pions



- Particle ratios as a function of S_{T}, S_0, and $S_0^{p_{\mathrm{T}}=1}$ show no variations
- Strangeness produces saturates after $N_{\mathrm{ch}}^{\mathrm{mid}} \geq 10$ → Event selection bias for S_{T}, S_0, and $S_0^{p_{\mathrm{T}}=1}$

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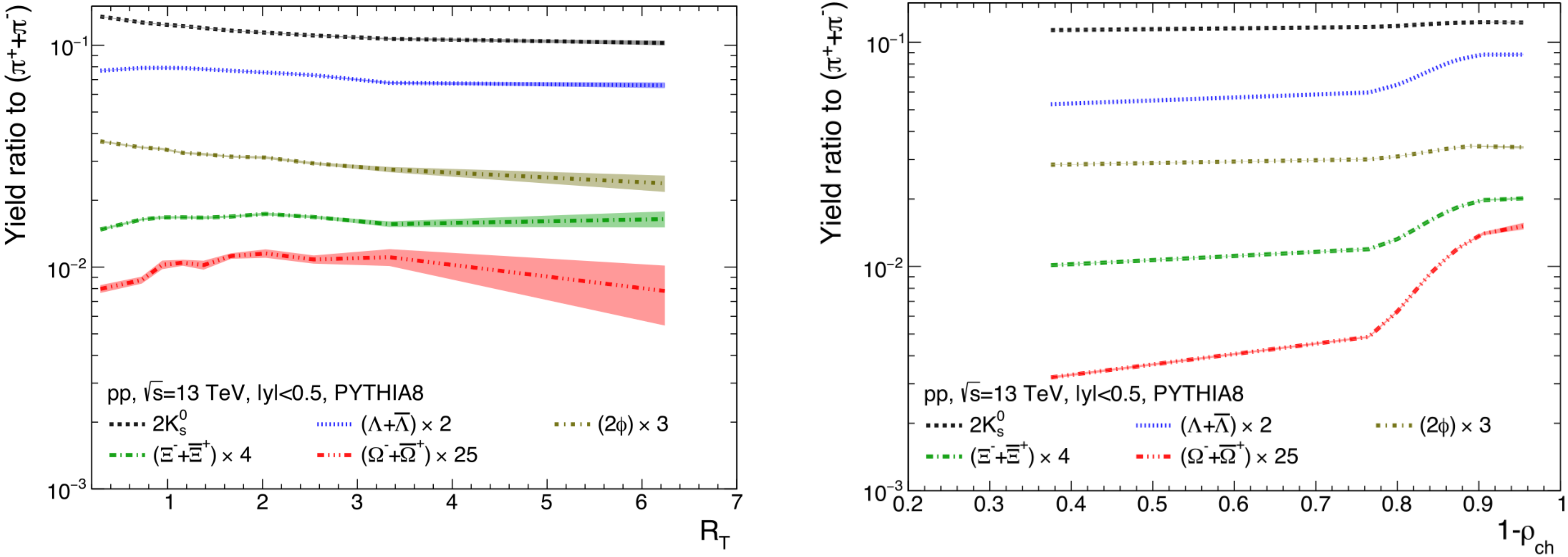
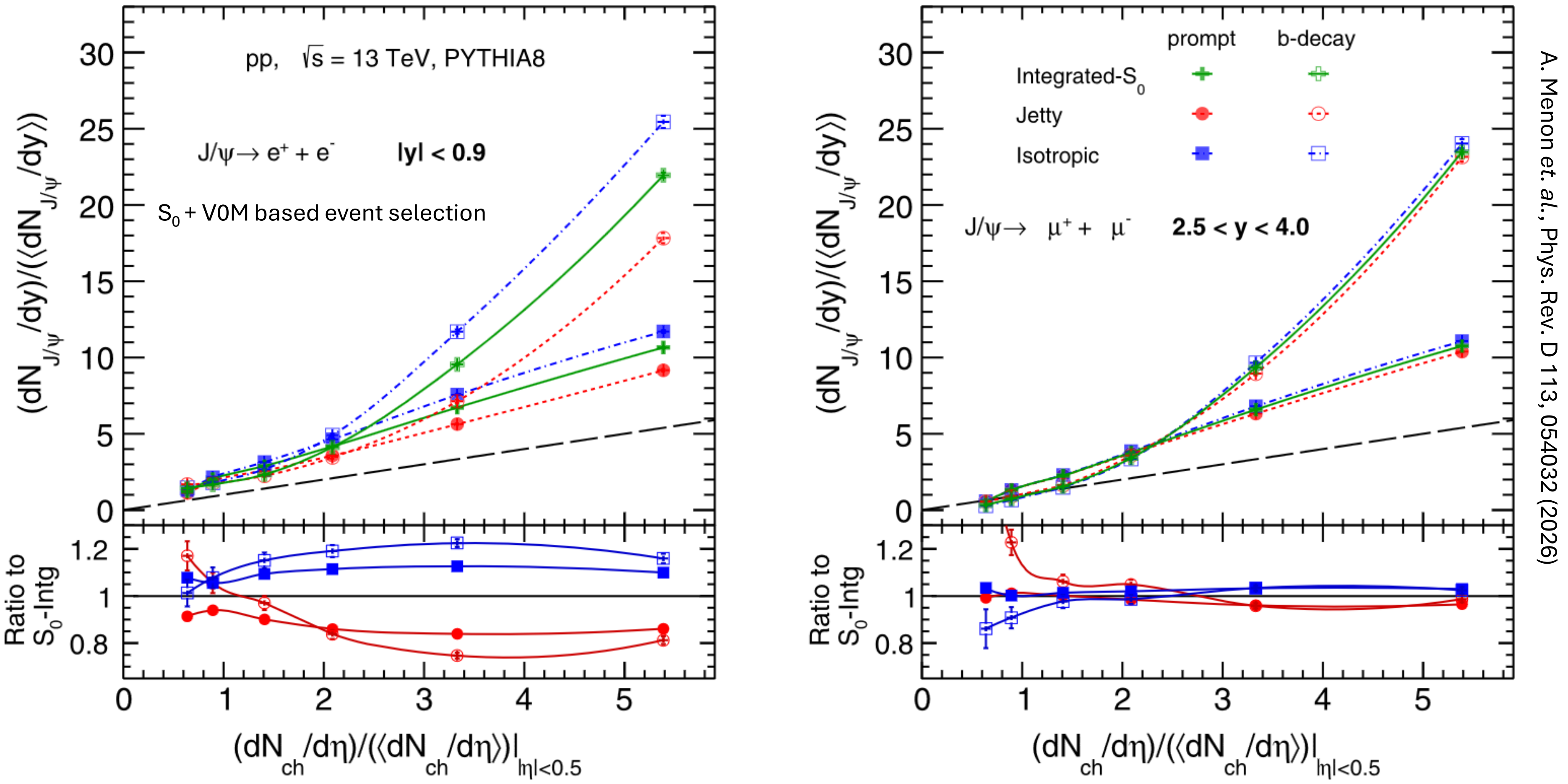
Particle yield ratios to charged pions



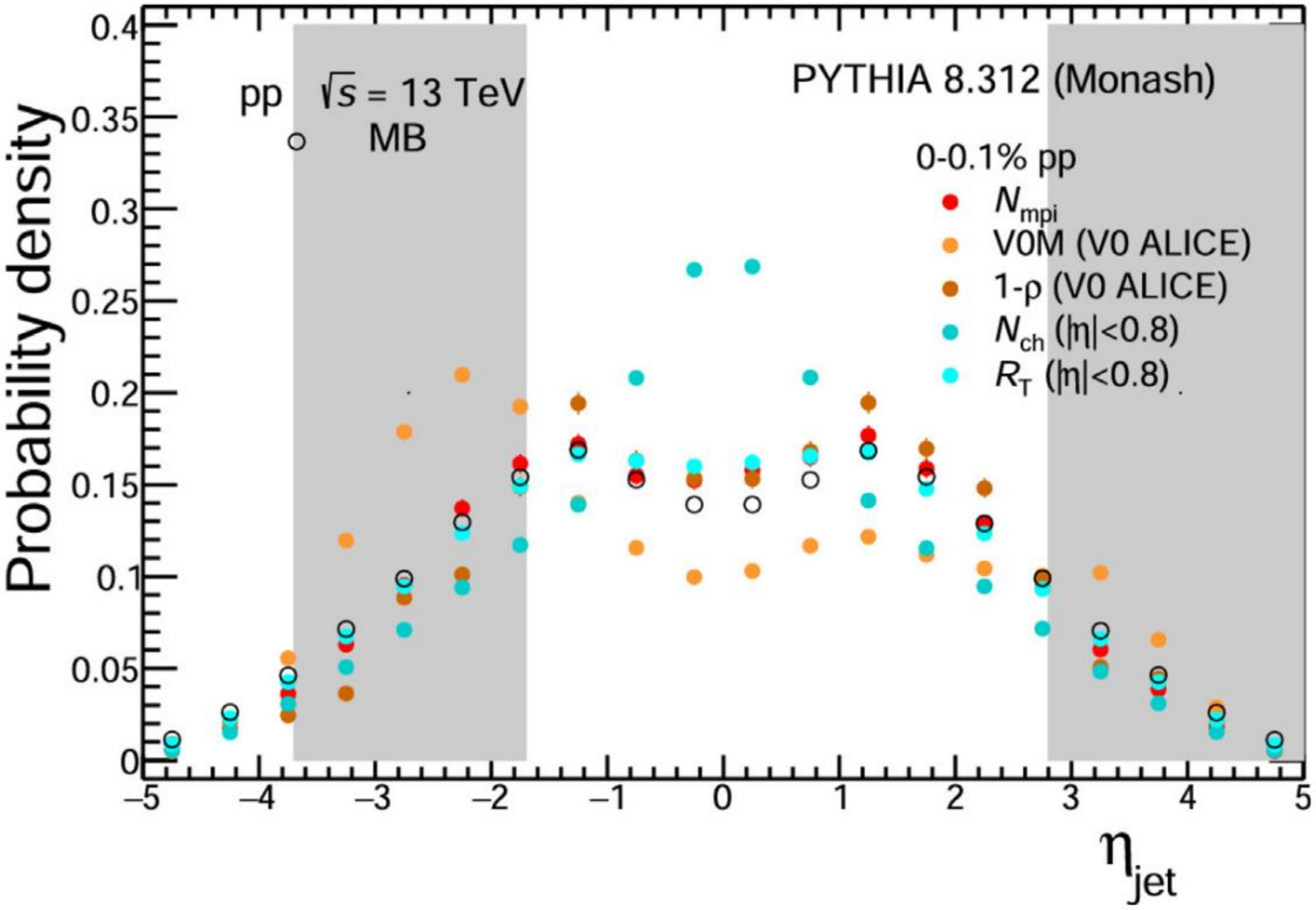
- R_{T} biases the sample with large number of soft charged particles (mostly pions) → Decreasing trend with R_{T}
- Omega has stronger N_{mpi} effects at lower R_{T} → Rising trend; Biasness takes over N_{mpi} at large R_{T}

Self normalized J/ψ Yield



- S_0-int : Nonprompt J/ψ yield is higher in forward rapidity than midrapidity
- S_0 selection has higher effect in J/ψ yield at midrapidity than forward rapidity

Recoil Jet η_{jet} Distribution



- Recoil jets reconstructed opposite to a high-p_T leading hadron
- N_{ch} and R_T show enhanced jet yields at midrapidity (|η|<0.8), indicating autocorrelation biases.
- V0M exhibits an asymmetric recoil-jet distribution due to its forward detector acceptance.
- Flattenicity closely reproduces N_{mpi} distributions, showing minimal selection bias.

Score Card

Estimator	Autocorr.	Jet bias	IRC safe	MPI sens.	Forward
$N_{\mathrm{ch}}^{\mathrm{mid}}$	🔴	🔴	—	🟢	🔴
$N_{\mathrm{ch}}^{\mathrm{fwd}}$	🟢	🟠	—	🟢	🟢
S_0 (spherocity)	🔴	🟠	🟢	🟠	🔴
S_{T} (sphericity)	🔴	🟢	🟢	🟠	🔴
R_{T} (UE activity)	🟠	🟢	—	🟢	🔴
ρ (flattenicity)	🟢	🟢	—	🟢	🟢

🟢 Good 🟠 Moderate 🔴 Problematic

Score Card

Estimator	Autocorr.	Jet bias	IRC safe	MPI sens.	Forward
$N_{\mathrm{ch}}^{\mathrm{mid}}$	🔴	🔴	—	🟢	🔴
$N_{\mathrm{ch}}^{\mathrm{fwd}}$	🟢	🟠	—	🟢	🟢
S_0 (spherocity)	🔴	🟠	🟢	🟠	🔴
S_{T} (sphericity)	🔴	🟢	🟢	🟠	🔴
R_{T} (UE activity)	🟠	🟢	—	🟢	🔴
ρ (flattenicity)	🟢	🟢	—	🟢	🟢

🟢 Good 🟠 Moderate 🔴 Problematic

IRC safety is not discussed explicitly in this talk.
Question: Do we require IRC safety for the kind of studies being discussed in this workshop?

What we learn?

- Event classifiers shape the physics message.
- Different estimators can change the apparent strength of QGP-like signals.
- Jet bias and autocorrelation can mimic QGP like signals
- Forward estimators reduce autocorrelation.
- Topology-aware estimators reduce hard-process bias.
- Multi-estimator consistency is the strongest cross-check.

A signal is trustworthy only when it survives careful control of the selection and the estimator.

How to Handle Biases?

- Keep the estimator and observable in separate rapidity regions whenever possible
- Perform double-differential analyses to disentangle multiplicity from topology.
- Report results with several estimators and flag any dependence on the choice.

Outlook

- Heavy flavour sector in small collision systems remain almost unexplored for the effects of event selection biases

- Apply event shape classifiers in heavy-ion collisions — Existing studies show exciting results!

- The new collision systems, pO, OO and Ne—Ne provide avenue to explore the onset of QGP like medium — This is where the event shape classifiers can fit in
  - ➢ The OO and Ne—Ne systems are comparable to p—Pb but having geometry and well defined impact parameter
  - ➢ Event-by-event geometry fluctuation remain major source of physics
  - ➢ Effect of clustered nuclear profile in final states ?? Can event shapes be useful to isolate the effects of nuclear geometry and their fluctuations from that of underlying event activity?

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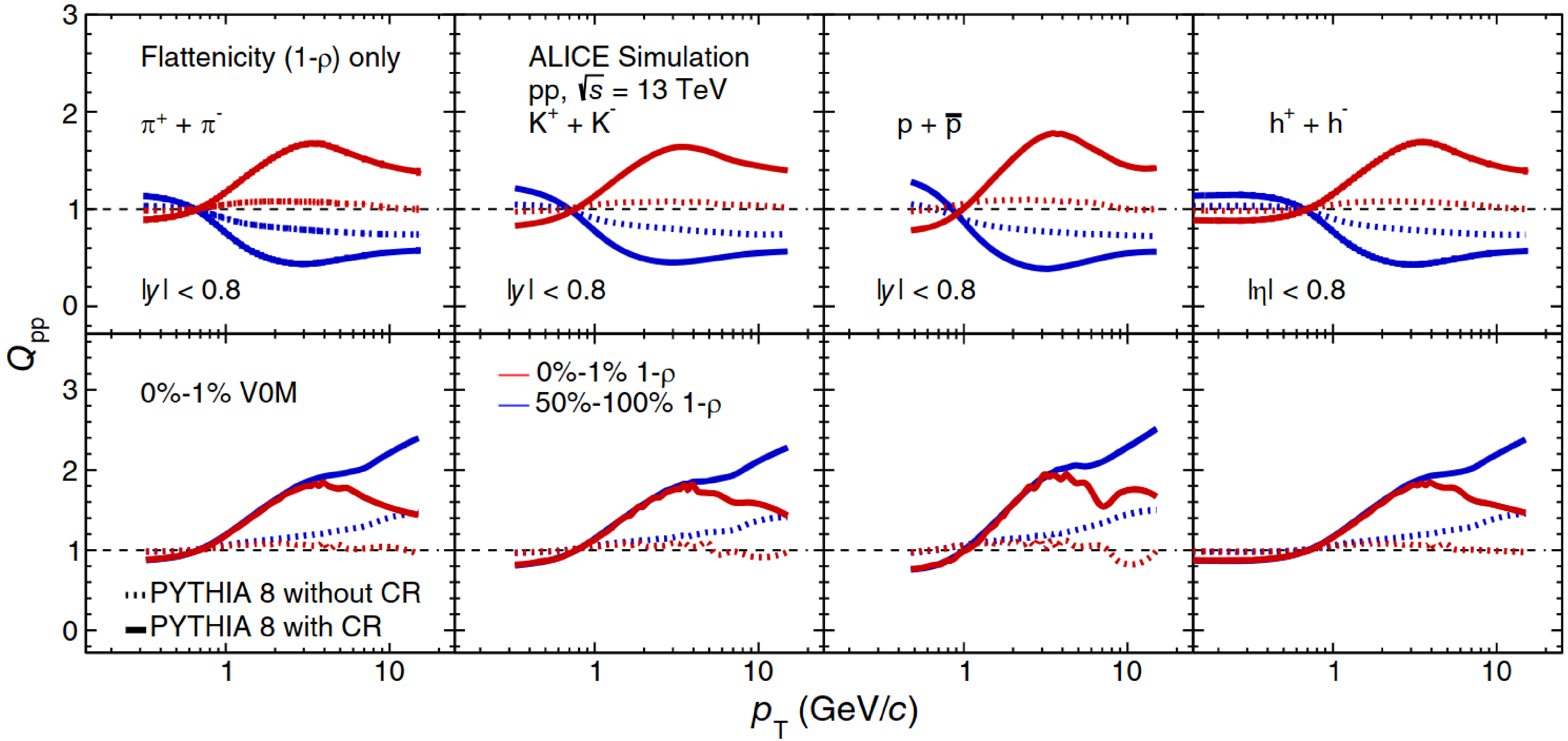
Backup

I acknowledge the support from the Hungarian National Research, Development and Innovation Office (NKFIH) under the contract numbers NKFIH NKKP ADVANCED_25-153456, 2025-1.1.5-NEMZ_KI-2025-00005, 2024-1.2.5-TET-2024-00022, and the usage of Wigner Scientific Computing Laboratory (WSCLAB)



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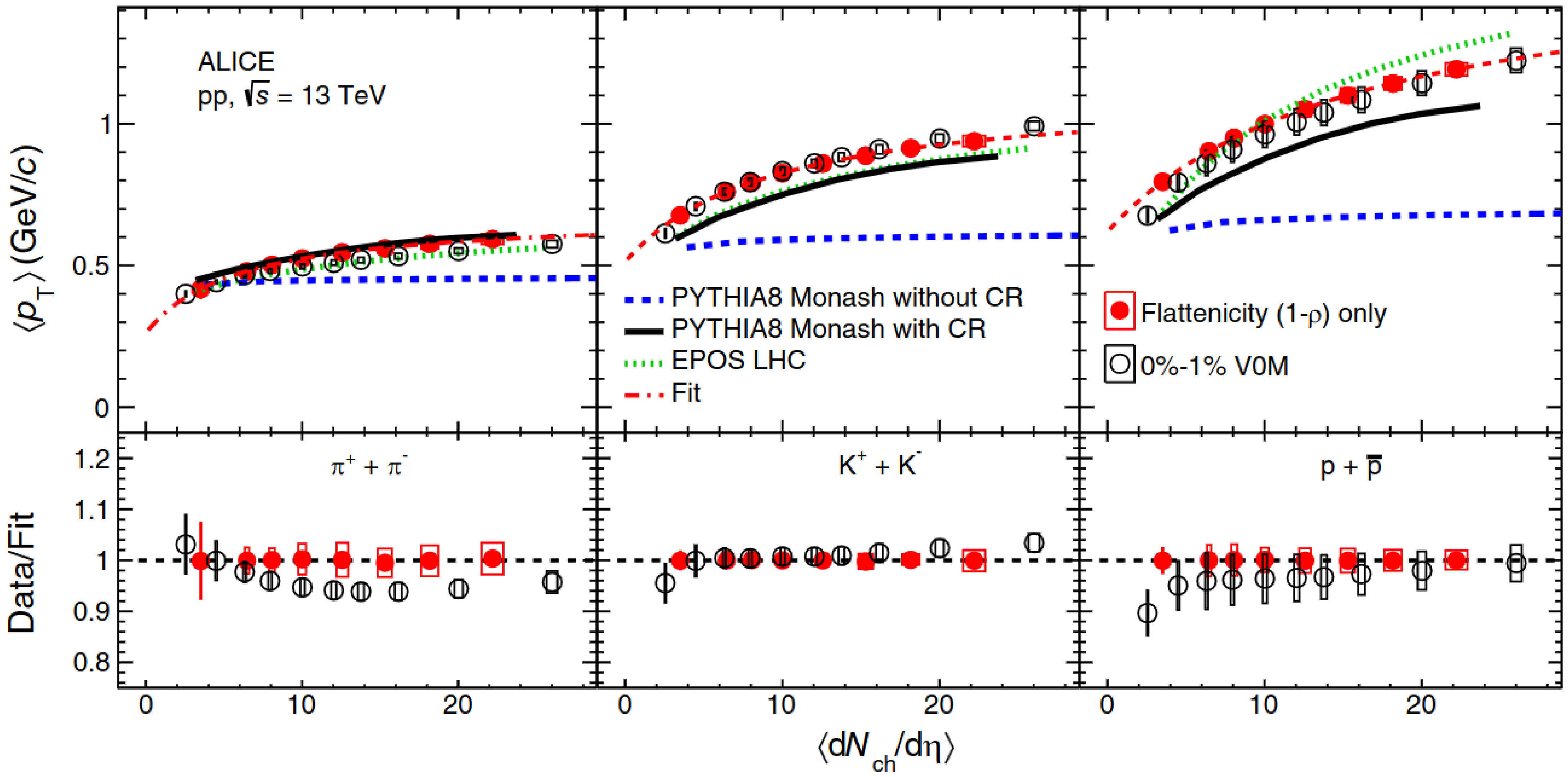

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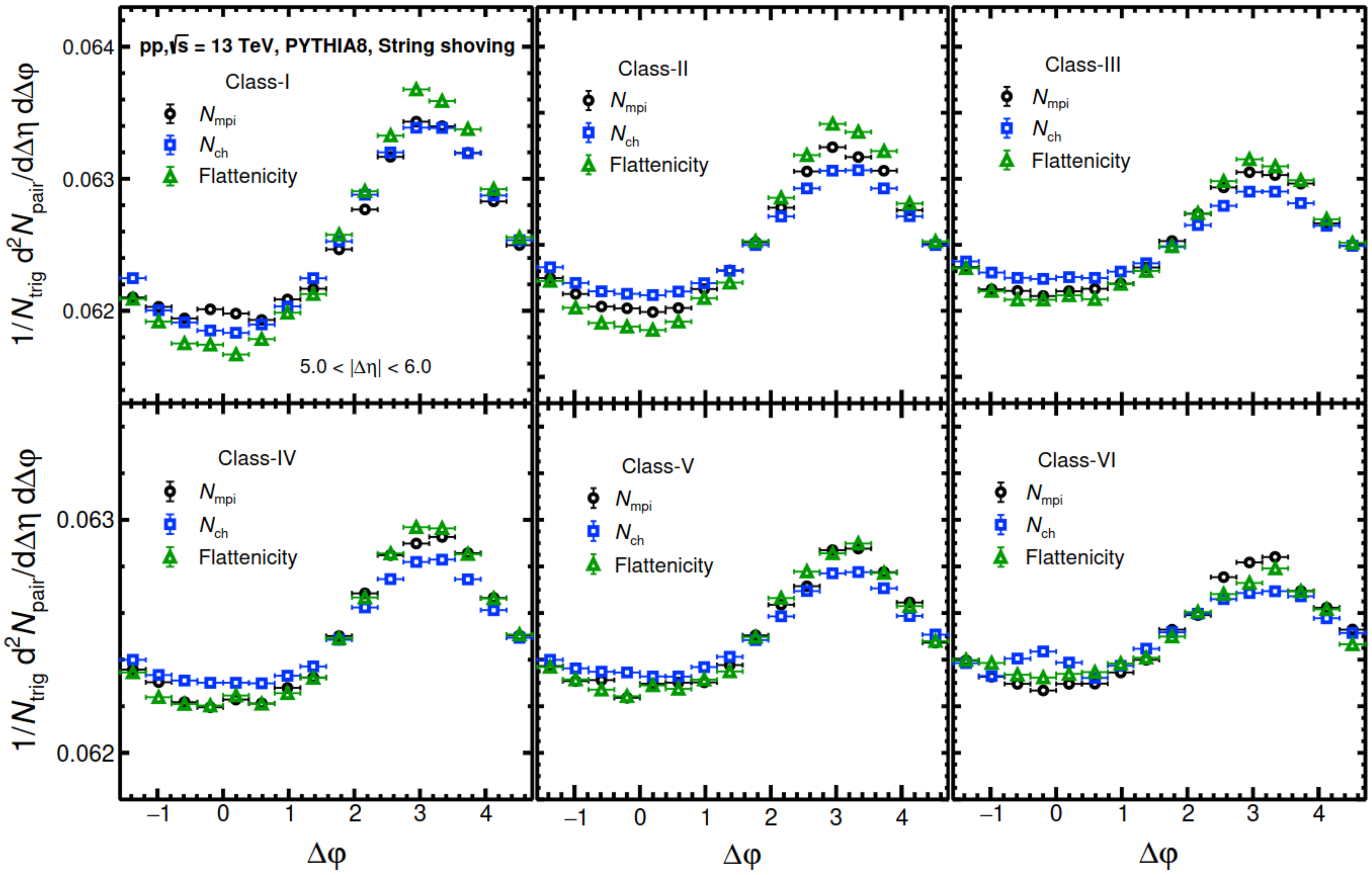

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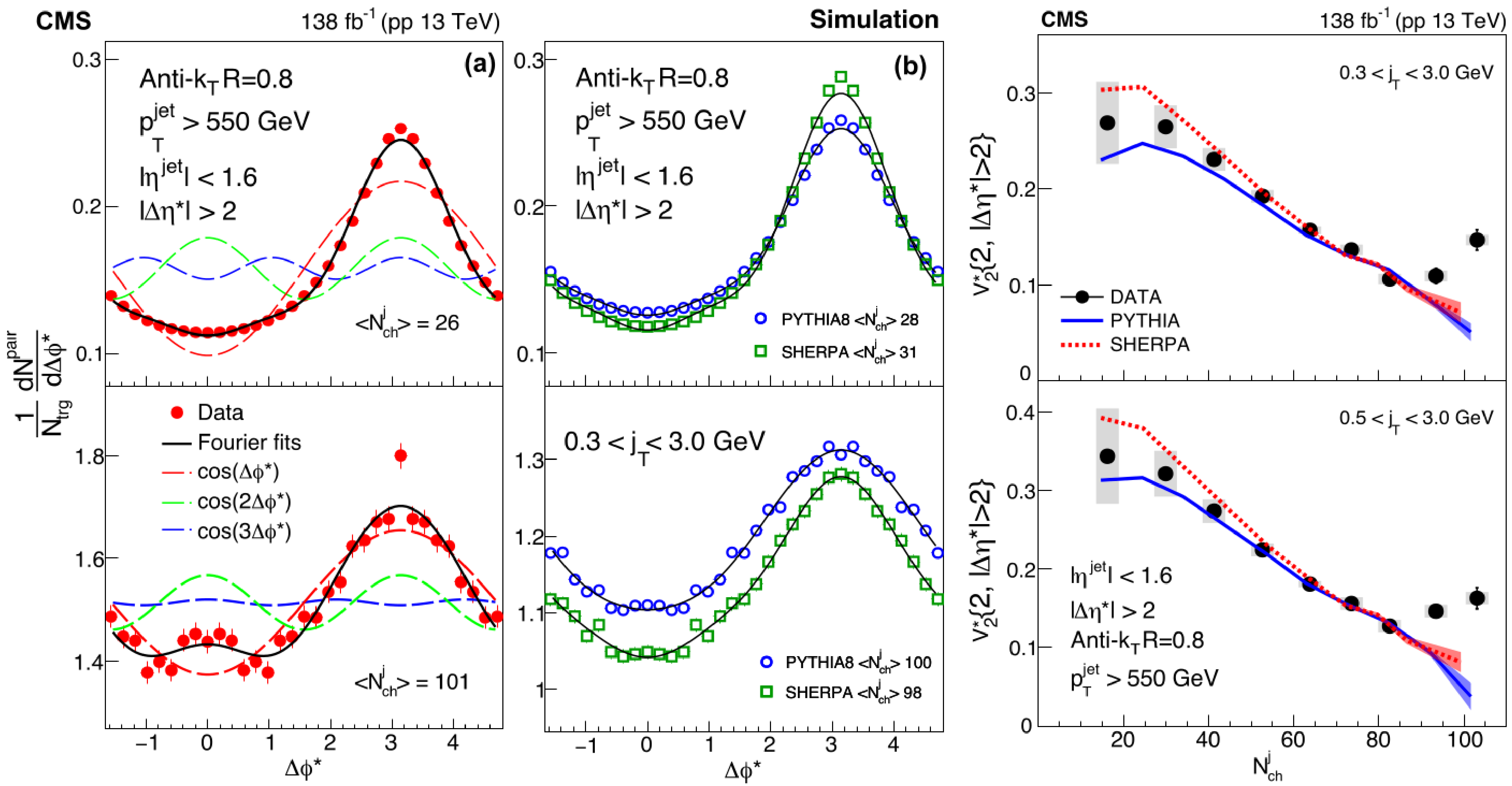
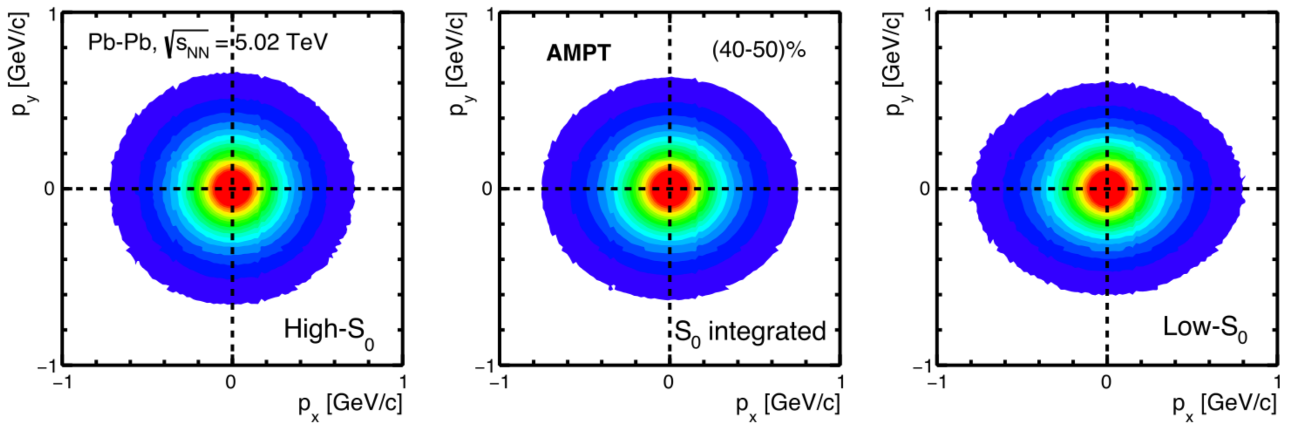


Phys. Rev. Lett. 133 (2024) 142301



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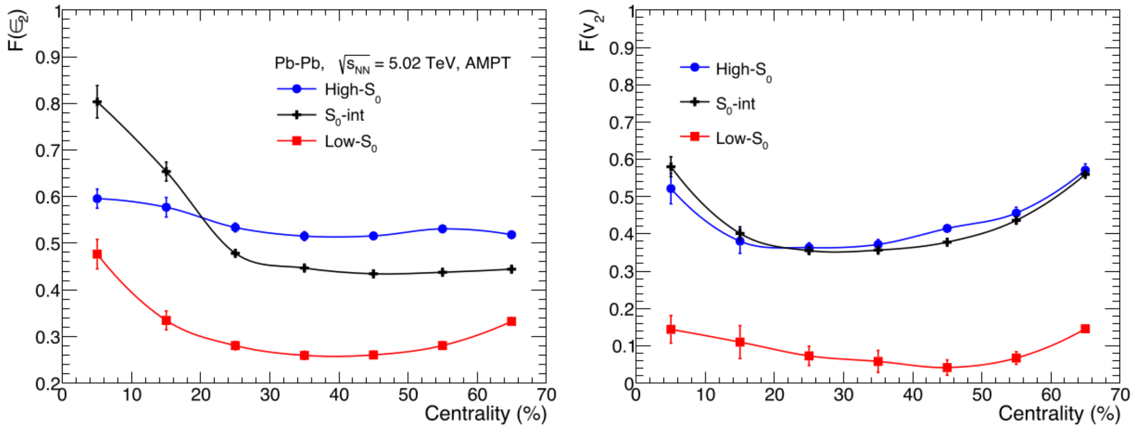

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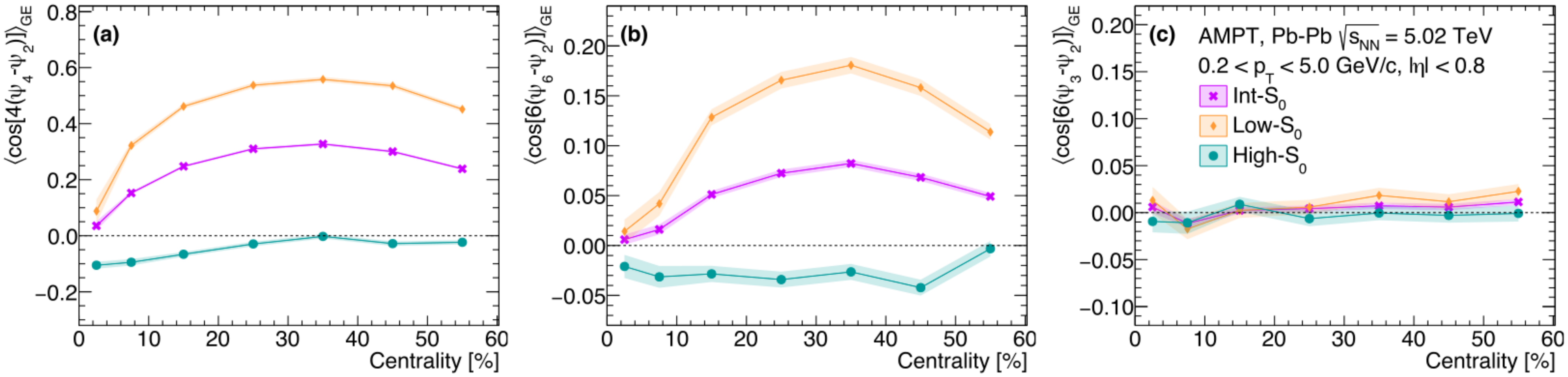

2.6.1. *Infrared safety*

In particle physics, infrared refers to the limit of the low-energy (large-wavelength) quanta, i.e., soft photons in QED and soft gluons in QCD. In high-energy collider experiments, soft gluons are frequently produced through parton radiation and showering. These gluons subsequently hadronize, giving rise to soft (low-p_{T}) hadrons in the final state. The invariance of a quantity in the presence of these soft hadrons is called infrared safety.

Let us consider a soft hadron with transverse momentum $\vec{p}_{\mathrm{T}_s}$ is emitted along with other hadrons such that $|\vec{p}_{\mathrm{T}_s}| = \varepsilon \to 0$. Here, the subscript '*s*' stands for soft. New value of transverse spherocity modifies to:

$$S_0' = \frac{\pi^2}{4} \min_{\hat{n}} \left(\frac{\sum_{i=1}^{N_{\mathrm{had}}} |\vec{p}_{\mathrm{T}_i} \times \hat{n}| + |\vec{p}_{\mathrm{T}_s} \times \hat{n}|}{\sum_{i=1}^{N_{\mathrm{had}}} |\vec{p}_{\mathrm{T}_i}| + |\vec{p}_{\mathrm{T}_s}|} \right)^2. \tag{12}$$

If the angle between $\vec{p}_{\mathrm{T}_s}$ and $\hat{n}$ is θ_s, then the above equation can be re-written as:

$$S_0' = \frac{\pi^2}{4} \min_{\hat{n}} \left(\frac{\sum_{i=1}^{N_{\mathrm{had}}} |\vec{p}_{\mathrm{T}_i} \times \hat{n}| + \varepsilon \sin\theta_s}{\sum_{i=1}^{N_{\mathrm{had}}} |\vec{p}_{\mathrm{T}_i}| + \varepsilon} \right)^2. \tag{13}$$

In the infrared limit, i.e., $\varepsilon \to 0$, Eq. (13) modifies to:

$$S_0' = \frac{\pi^2}{4} \min_{\hat{n}} \left(\frac{\sum_{i=1}^{N_{\mathrm{had}}} |\vec{p}_{\mathrm{T}_i} \times \hat{n}|}{\sum_{i=1}^{N_{\mathrm{had}}} |\vec{p}_{\mathrm{T}_i}|} \right)^2 = S_0. \tag{14}$$

Thus, a soft (infrared) emission of particles renders transverse spherocity unchanged, making it infrared safe.

2.6.2. Collinear safety

Let us consider a hadron with transverse momentum p_{T_k} splits into two collinear particles with transverse momenta $p_{\mathrm{T}_{k1}}$ and $p_{\mathrm{T}_{k2}}$, such that:

$$p_{\mathrm{T}_{k1}} = z p_{\mathrm{T}_k} \quad \text{and} \quad p_{\mathrm{T}_{k2}} = (1-z) p_{\mathrm{T}_k}. \tag{15}$$

Here, z and $(1-z)$ stand for the fractions of the momentum of the hadron in a collinear splitting and $0 < z < 1$ and $\vec{p}_{\mathrm{T}_k}$, $\vec{p}_{\mathrm{T}_{k1}}$, and $\vec{p}_{\mathrm{T}_{k2}}$ are parallel to each other. For any direction of $\hat{n}$,

$$|\vec{p}_{\mathrm{T}_{k1}} \times \hat{n}| + |\vec{p}_{\mathrm{T}_{k2}} \times \hat{n}| = z|\vec{p}_{\mathrm{T}_k} \times \hat{n}| + (1-z)|\vec{p}_{\mathrm{T}_k} \times \hat{n}| = |\vec{p}_{\mathrm{T}_k} \times \hat{n}|. \tag{16}$$

Thus, a collinear splitting of a particle keeps the numerator of Eq. (11) unchanged. Similarly, one can show for the denominator as follows.

$$|\vec{p}_{\mathrm{T}_{k1}}| + |\vec{p}_{\mathrm{T}_{k2}}| = z|\vec{p}_{\mathrm{T}_k}| + (1-z)|\vec{p}_{\mathrm{T}_k}| = |\vec{p}_{\mathrm{T}_k}|. \tag{17}$$

12